[lex179] Differential recursion relation for Poisson and Pascal statistics

(a) Show that the Poisson and Pascal probability distributions,

$$P_n(t) = \frac{(t/\tau)^n}{(1+t/\tau)^{n+1}}, \qquad P_n(t) = \frac{(t/\tau)^n}{n!} e^{-t/\tau},$$

satisfy the differential recursion relation,

$$(n+1)P_{n+1}(t) = nP_n(t) - tP_n'(t), \quad n = 1, 2, \ldots$$

(b) Show that the above recursion relation is equivalent to the relation,

$$P_n(t) = \frac{(-t)^n}{n!} \frac{d^n}{dt^n} P_0(t), \quad n = 1, 2, \ldots$$

for any differentiable function $P_n(t)$.

Solution: