[lex178] Super-Poisson interpolation statistics II

Consider the super-Poisson probability distribution,

$$P_n(t) = \left(1+\frac{1}{m}\right)\left(1+\frac{2}{m}\right)\cdots\left(1+\frac{n-1}{m}\right)\frac{1}{n!}\left(\frac{t}{\tau}\right)^n\left(1+\frac{t}{m\tau}\right)^{-(m+n)},$$

established in [lex177].
(a) Show that this distribution is properly normalized:

$$\sum_{n=0}^{\infty} P_n(t) = 1 \quad : \; t \geq 0.$$

(b) Show that the mean value $\langle n\rangle$ and the variance $\langle\langle n^2\rangle\rangle$ defined as

$$\langle n\rangle \doteq \sum_{n=0}^{\infty} nP(n), \quad \langle n^2\rangle \doteq \sum_{n=0}^{\infty} n^2 P(n), \quad \langle\langle n^2\rangle\rangle \doteq \langle n^2\rangle - \langle n\rangle^2$$

produce the following results:

$$\langle n\rangle = \frac{t}{\tau}, \quad \langle\langle n^2\rangle\rangle = \frac{t}{\tau}\left(1+\frac{t}{m\tau}\right).$$

Solution: