[lex175] Bloch equations II

Consider the Bloch equations,

$$\frac{dM_x}{dt} = \gamma[M_yB_z - M_zB_y] - \frac{M_x}{T_2},$$
$$\frac{dM_y}{dt} = \gamma[M_zB_x - M_xB_z] - \frac{M_y}{T_2},$$
$$\frac{dM_z}{dt} = \gamma[M_xB_y - M_yB_x] + \frac{M_{\rm eq} - M_z}{T_1},$$

for the relaxation of the vector $\mathbf{M} = (M_x, M_y, M_z)$, where γ is the gyromagnetic ratio, T_2 the spin-spin relaxation time, T_1 the spin-lattice relaxation time, and $\mathbf{B} = (B_x, B_y, B_z)$ a magnetic field with dominant z-component. The equilibrium magnetization is governed by the static susceptibility via the relation, $M_{\rm eq} = \chi(T)B_z$. In [lex174] we established the analytic solution for the case $B_x = B_y = 0$. Its long-time asymptotic values are $M_x \to 0$, $M_y \to 0$, and $M_z \to M_{\rm eq}$.

(a) Solve the Bloch equations numerically for the case $B_x = 0$, $B_y = B_z = 20$, $\gamma = 1$, $T_1 = 2$, $T_2 = 0.5$, and initial condition $M_x(0) = M_y(0) = 0$, $M_z(0) = M_{\rm eq} = 0.3$. Plot the following quantities versus $0 < t < 2$: M_x, M_y, M_z, $M_\perp \doteq \sqrt{M_x^2 + M_y^2}$, $|M| \doteq \sqrt{M_x^2 + M_y^2 + M_z^2}$. Interpret your observations.

Initial conditions that differ from the equilibrium conditions of the case $B_x = B_y = 0$ analyzed in [lex174] can be established by adding to the constant B_z a pulse of B_y of short duration as is evident from the results of part (a).

(b) Identify the first maximum value of $M_\perp$ and the time at which it is realized. That time would be chosen as the duration of the pulse for the purpose of producing initial conditions with nonzero transverse magnetization.

Solution: