[lex174] Bloch equations I

Consider the Bloch equations,

$$\frac{dM_x}{dt} = \gamma[M_y B_z - M_z B_y] - \frac{M_x}{T_2},$$
$$\frac{dM_y}{dt} = \gamma[M_z B_x - M_x B_z] - \frac{M_y}{T_2},$$
$$\frac{dM_z}{dt} = \gamma[M_x B_y - M_y B_x] + \frac{M_{\rm eq} - M_z}{T_1},$$

for the relaxation of the vector $\mathbf{M} = (M_x, M_y, M_z)$, where γ is the gyromagnetic ratio, T_2 the spin-spin relaxation time, T_1 the spin-lattice relaxation time, and $\mathbf{B} = (B_x, B_y, B_z)$ a magnetic field with dominant z-component. The equilibrium magnetization is governed by the static susceptibility via the relation, $M_{\rm eq} = \chi(T)B_z$.
(a) Find the analytic solution of the Bloch equation for the case $B_x = B_y = 0$ and initial conditions $M_x(0) = 0$, $M_y(0) = M_\perp^0$, and $M_z(0) = M_z^0$.
(b) Plot $M_x(t)$ and $M_y(t)$ versus $0 < t < 3$ for $\gamma = 1$, $B_z = 20$, $T_2 = 0.5$, and $M_\perp = 1$.
(c) Plot $M_z(t)$ versus $0 < t < 10$ for $\gamma = 1$, $T_1 = 2$, and $M_z^0 = 0.9$, $M_z^0 = -0.5$ (two curves).

The components M_x and M_y describe damped oscillations with initial amplitude $M_\perp^0$ and decay rate T_2^{-1}. The component M_z decays uniformly from the initial value M_z^0 toward the equilibrium value $M_{\rm eq}$ at the rate T_1^{-1}. The magnitude of the vector $\mathbf{M}$ is conserved only for $T_1 \to \infty$, $T_2 \to \infty$.

Establishing initial conditions that differ from the equilibrium conditions, i.e. from long-time asymptotic solutions of the Bloch equations, are described in [lex175].

Solution: