[lex172] Phase velocity and group velocity

Electromagnetic waves constrained by wave guides are, in general, dispersive, thus characterized by distinct phase and group velocities:

$$v_{\mathrm{ph}} \doteq \frac{\omega}{k}, \quad v_{\mathrm{gr}} \doteq \frac{d\omega}{dk}.$$

Find the general structure of the dispersion $\omega(k)$ that produces the following relations between phase velocity v_{ph}, group velocity v_{gr}, and speed of light in vacuum c:
(a) $v_{\mathrm{ph}} v_{\mathrm{gr}} = c^2$, (b) $v_{\mathrm{ph}}^2 v_{\mathrm{gr}} = c^3$, (c) $v_{\mathrm{ph}} v_{\mathrm{gr}}^2 = c^3$.

Solution: