[lex171] Rate of energy absorption in NMR

Consider a sample of $N = N_+(t) + N_-(t)$ nuclear spins $I = \frac{1}{2}$, an ensemble of two level systems with a gap of size $E_+ - E_- = \hbar\omega$ as described in [lln23]. The spin-lattice relaxation process during irradiation is governed by linear ODE,

$$\frac{d}{dt}n(t) = -2Wn(t) + \frac{n_{\rm eq} - n(t)}{T_1}, \tag{1}$$

for the excess population of the lower level, $n(t) \doteq N_-(t) - N_+(t)$, where W is the transition rate and $n_{\rm eq}$ the equilibrium value of the excess population.
(a) Determine the solution (i) for initial condition $n(0) = 0$ and (ii) $n(0) = n_{\rm eq}$.
(b) Show that the rate of energy absorption in the face of simultaneous irradiation and spin-lattice relaxation is

$$\left.\frac{dE}{dt}\right|_{\rm abs} = \hbar\omega W n(t), \quad E(t) \doteq N_-(t)E_- + N_+(t)E_+. \tag{2}$$

(c) Identify the long-time asymptotic behavior of this absorption rate and discuss the governing time scale at low and high intensity of radiation.
Note: Only the first term on the right-hand side of (1) contributes to energy absorption.

Solution: