[lex17] Conducting sphere in uniform electric field

A grounded conducting sphere of radius R is placed into a region where a uniform electric field $\mathbf{E}_{\text{ap}} = E_0\,\hat{\mathbf{k}}$ is present. Electrostatic equilibrium is restored by a nonuniform surface charge density σ, which modifies the electric field surrounding the sphere.
(a) Show that an electric potential of the form,

$$\Phi(r,\theta) = \frac{a}{r} + b + \frac{c\cos\theta}{r^2} + dr\cos\theta, \tag{1}$$

(in spherical coordinates) is a solution of the Laplace equation.
(b) Determine the coefficients a, b, c, d in (1) such that physically meaningful boundary conditions at $r = R$ and at $r = \infty$ are satisfied.
(c) Determine the surface charge density $\sigma(R,\theta)$.
(d) One hemisphere of the conducting surface will be positively charged and the other negatively by an equal amount. Find the induced charge Q_{hs} on each hemisphere.
(e) The applied field $\mathbf{E}_{\text{ap}}$ induces an electric dipole on the sphere, which is reflected in the third term of (1). Find the induced electric dipole moment $\mathbf{p}$ for this situation.
(f) If the grounded sphere is replaced a conducting sphere that carries a charge Q_0 on its surface, what are the coefficients a, b, c, d in (1) for that case?

Solution: