[lex169] Lorentz invariance of momentum conservation

Two particles with equal rest masses m are undergoing an inelastic collision as shown in the lab frame S. Conservation of total momentum implies

$$\tilde{m}(v)\mathbf{v} + \tilde{m}(0)0 = \tilde{M}(\bar{v})\bar{\mathbf{v}},$$

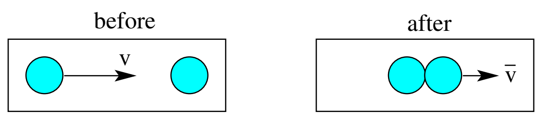
where $v = 2\bar{v}/(1+\bar{v}^2/c^2)$ from the relativistic velocity addition rule as shown in [lln16] and $\tilde{m}(v)$ is the relativistic mass to be determined. Use the requirement that the transverse component of the total momentum must be conserved in frame S'' which moves with relative velocity $\mathbf{u}$ perpendicular to $\mathbf{v}$ to infer the relation,

$$\tilde{m}(v'')\mathbf{u} + \tilde{m}(u)\mathbf{u} = \tilde{M}(\bar{v}'')\mathbf{u},$$

where $v'' = \sqrt{v^2 + u^2(1 - v^2/c^2)}$ and $\bar{v}'' = \sqrt{\bar{v}^2 + u^2(1 - \bar{v}^2/c^2)}$. In the limit $u \to 0$, this becomes the relativistic relation,

$$\tilde{m}(v) + \tilde{m}(0) = \tilde{M}(\bar{v}),$$

between the individual masses before the collision and composite mass after the collision.



Solution: