[lex16] Point charge near perpendicular plane conducting surfaces

Consider two plane conducting surfaces at right angle. The conducting material is at $y < 0$ and at $z < 0$. Now we place a point charge $q > 0$ at $x = 0$, $y = y_0 > 0$, $z = z_0 > 0$ outside the conductor and investigate the electrostatic equilibrium. For this purpose we place one positive and two negative image point charges such that the electric potential potential $\Phi(\mathbf{x})$ of the four point charges combined vanishes at both surfaces. We use symmetry as our guiding principle.
(a) Calculate the electric field $\mathbf{E} = -\nabla\Phi(\mathbf{x})$ and verify that the two tangential components vanish at each surface.
(b) Infer from the normal component of the electric field at each surface the surface charge densities,

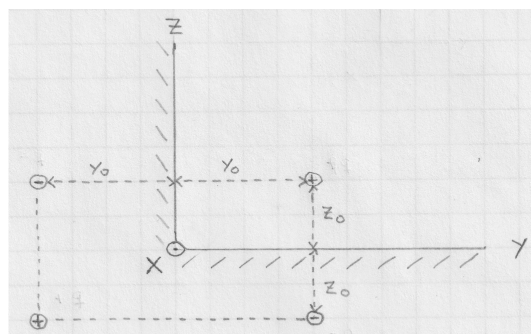
$$\sigma_{xz}(x,z) = \epsilon_0 E_y(x,0,z) \quad : \; z > 0; \qquad \sigma_{xy}(x,y) = \epsilon_0 E_z(x,y,0) \quad : \; y > 0.$$

Graphically represent the profile of the induced charge densities in the form of contour plots.
(c) Determine via integration the total charge on each surface:

$$q_{xz} = \int_{-\infty}^{+\infty} dx \int_0^{\infty} dz \sigma_{xz}(x,z), \quad q_{xy} = \int_{-\infty}^{+\infty} dx \int_0^{\infty} dy \sigma_{xy}(x,y).$$

Show that the induced charge on the two surfaces is

$$Q_{\text{ind}} = q_{xz} + q_{xy} = -\frac{2q}{\pi}\left[\arctan\frac{y_0}{z_0} + \arctan\frac{z_0}{y_0}\right] = -q.$$



Solution: