[lex158] Einstein coefficient for hydrogen $2s \to 3p$ transition

Identify the (real) electronic wave functions for the $2s$ and $3p$ states with $m = 0$ of the hydrogen atom and express them as functions of the spherical variables r, θ with the Bohr radius a_0 as the only parameter. Transitions with $\Delta m = 0$ are not prohibited if the light is polarized in z-direction.
(a) Check the normalization of both wave functions, named $\psi_1(r,\theta)$ and $\psi_2(r,\theta)$ for the two resonant levels under consideration here.
(b) Calculate the electric-dipole matrix element,

$$\mu_{12} \doteq -e \int d^3r\, \psi_1^*(\mathbf{r}) z \psi_2(\mathbf{r}),$$

for the $2s \to 3p$ transition as a function of a_0 and the elementary charge e.
(c) Calculate the numerical value (in SI units) of the Einstein coefficient, using the expression,

$$B_{12} = \frac{\pi \mu_{12}^2}{3\epsilon_0 \hbar^2},$$

determined in [lln27].

Solution: