

[lex156] Rabi oscillations off resonance

The coupled ODEs for the coefficients of the coherent superposition state of a two level system subject to high-intensity light slightly off resonance (see [ln27]) can be simplified into

$$\dot{c}_1(t) = \frac{i}{2} \Omega_R e^{i\delta\omega t} c_2(t), \quad \dot{c}_2(t) = \frac{i}{2} \Omega_R e^{-i\delta\omega t} c_1(t); \quad \delta\omega \doteq \omega - \omega_0, \quad \Omega_R = \left| \frac{\mu E_0}{\hbar} \right|.$$

Extract a second-order linear ODE for $c_2(t)$ and solve it by invoking the ansatz: $c_2(t) = C e^{-i\zeta t}$. Then proceed with that solution and appropriate boundary conditions to show that the modified transition rate becomes

$$|c_2(t)|^2 = \left(\frac{\Omega_R}{\Omega} \right)^2 \sin^2 \left(\frac{\Omega t}{2} \right), \quad \Omega^2 = \Omega_R^2 + (\delta\omega)^2.$$

Solution: