[lex154] Coherent state of quantum harmonic oscillator I

The pure one-parameter quantum state,

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle,$$

where α is the complex-valued parameter, is known as a coherent state of the quantum harmonic oscillator used for modeling a stream of photons [lln26].
(a) Show that the state $|\alpha\rangle$ satisfies the eigenvalue equations, $a|\alpha\rangle = \alpha|\alpha\rangle$ and $\langle\alpha|a^\dagger = \langle\alpha|\alpha^*$.
(b) Show that the state $|\alpha\rangle$ is normalized.
(c) Show that the average number of photons in the state $|\alpha\rangle$ is $\langle n\rangle = |\alpha|^2$.
(d) Show that the variance in the number of photons is equal to its mean for the state $|\alpha\rangle$.
(e) Show that the probability $|\langle n|\alpha\rangle|^2$ for the state $|\alpha\rangle$ to contain n photons is determined by the Poisson distribution.

Solution: