[lex153] Photon statistics from Planck's radiation law

Planck's expression for the spectral energy density in a cavity at thermal equilibrium reads [lln24]:

$$u(\omega) = \frac{\omega^2}{\pi^2 c^3} \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1}.$$

Use the thermodynamic relation relating fluctuations of internal energy to heat capacity [tex109],

$$\langle\langle U^2 \rangle\rangle = k_{\mathrm{B}} T^2 C_V, \quad C_V = \left(\frac{\partial U}{\partial T}\right)_V,$$

to derive the relation, $\langle\langle n^2 \rangle\rangle = \langle n \rangle + \langle n \rangle^2$, characteristic of Pascal statistics.

Solution: