[lex151] Erlang distributions I

Consider the (continuous) Erlang distribution for index $k = 2$,

$$f(t) \doteq \frac{4t}{\tau^2} e^{-2t/\tau}.$$

Prepare a Mathematica notebook or equivalent to carry out the following list of tasks:
(a) Show that this function is normalized and that its mean is equal to τ.
(b) Infer from $f(t)$ a discrete probability distribution $P_n(t)$ via integration and the use of a recursion relation as follows:

$$P_0(t) = \int_t^\infty dt' f(t'), \quad P_n(t) = \int_0^t dt' f(t') P_{n-1}(t - t').$$

(c) Show that this (discrete) probability distribution is normalized as well.
(d) Calculate the mean $\langle n \rangle$ and the variance $\langle\langle n^2 \rangle\rangle$.
(e) Plot $P_n(t)$ as a function of t/τ for increasing values of n.

Solution: