

[lex150] Mean and variance of Pascal distribution

The Pascal distribution (also named geometric distribution),

$$P(n) = (1 - \gamma)\gamma^n \quad : \quad n = 0, 1, 2, \dots,$$

is a probability distribution with many different applications including the level occupancies of a quantum harmonic oscillator. Use elementary means to show that

(a) the distribution is properly normalized,

(b) the mean is $\langle n \rangle = \gamma/(1 - \gamma)$,

(c) the variance is $\langle \langle n^2 \rangle \rangle = \langle n \rangle + \langle n \rangle^2$.

Solution: