[lex146] Fields between moving capacitor plates II

Two oppositely charged conducting plates are placed in a coordinate system as shown. In the rest frame $\mathcal{F}$ of the plates, there is a uniform electric field $\mathbf{E} = E_0\,\hat{\mathbf{k}}$ between the plates. The Lorentz transformation predicts that in the frame $\mathcal{F}'$, which moves with velocity $\mathbf{v} = v\,\hat{\mathbf{i}}$ relative to $\mathcal{F}$, the electric field is stronger and there is is also a magnetic field:

$$\mathbf{E}' = \gamma E_0\,\hat{\mathbf{k}}, \quad \mathbf{B}' = \frac{\beta\gamma}{c}\,E_0\,\hat{\mathbf{j}}, \quad \beta \doteq \frac{v}{c}, \quad \gamma \doteq \frac{1}{\sqrt{1-\beta^2}}.$$

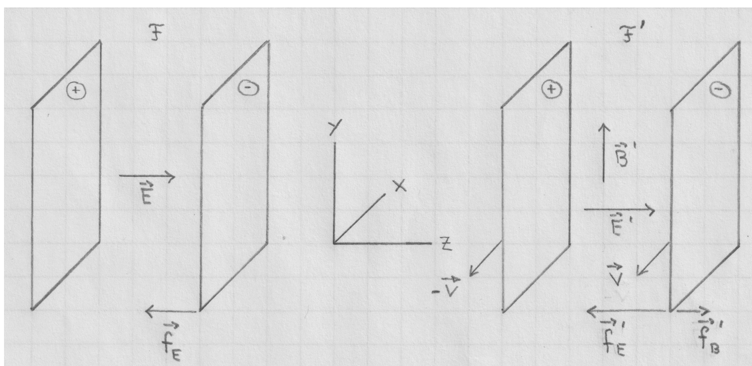
In frame $\mathcal{F}$ there is energy density u and zero energy current density $\mathbf{S}$. In frame $\mathcal{F}'$ there is a higher energy density u' and a nonzero energy current density $\mathbf{S}'$.

(a) Calculate u and u' as functions of $\epsilon_0, E_0, \beta, \gamma$.

(b) Calculate magnitude (as a function of $\epsilon_0, v, \gamma, E_0$) and direction of $\mathbf{S}'$.

(c) In a plane electromagnetic wave, all energy travels at the speed of light. This fact is encapsulated in the relation $|\mathbf{S}|/c = u$ between Poynting vector and energy density [lln16]. Show that the relation between Poynting vector and energy density for the situation investigated here reads

$$\beta\,\frac{|\mathbf{S}'|}{c} = u' - u.$$



Solution: