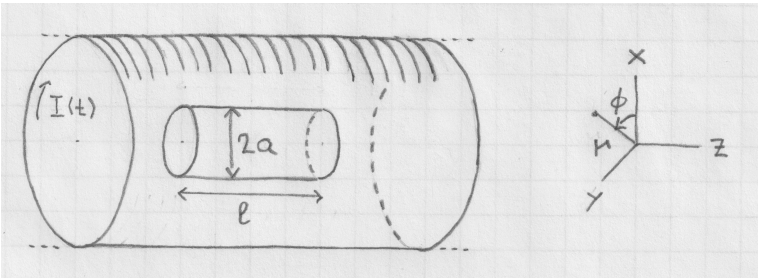
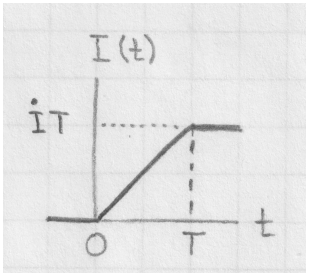
[lex143] Field energy inside solenoid I

Consider a long solenoid with a winding of n turns per unit length in the process of a steady buildup of current $I(t) = \dot{I}t$ with constant $\dot{I}$, beginning at $t = 0$ and ending at $t = T$. Near the axis of the solenoid, inside the cylinder of length l and radius a, a uniform magnetic field $\mathbf{B}(t)$ is being generated by the current.
(a) Express $\mathbf{B}(t)$ during current buildup in cylindrical coordinates as a function of $\dot{I}, n, t$.
(b) Calculate the magnetic-field energy U_B inside the cylinder as a function of $\dot{I}, T, n, a, l$ once the current is fully built up.
(c) During the current buildup, a steady electric field $\mathbf{E}$ with a radial profiles is present as well inside the cylinder. Express $\mathbf{E}$ in cylindrical coordinates as a function of $\dot{I}, n, r$.
(d) Determine the Poynting vector $\mathbf{S}(t)$ associated with the two fields during current buildup and infer from the result the direction field energy enters the cylinder.
(e) Calculate the flux $\Phi_S(t)$ of energy current density $\mathbf{S}(t)$ across the surface of the cylinder. Then integrate that quantity over the time of the charging process to recover U_B from part (b).
(f) Calculate the (time-independent) electric field energy U_E present inside the cylinder during current buildup. Express U_B as a function of $a, l, \dot{I}, n$.
(g) Show that the ratio U_E/U_B of the electric field energy during current buildup and magnetic field energy after after current buildup is a function of a/cT for the process described.
Note: Most expressions also contain factors of ϵ_0, μ_0, or $c = 1/\sqrt{\epsilon_0\mu_0}$.





Solution: