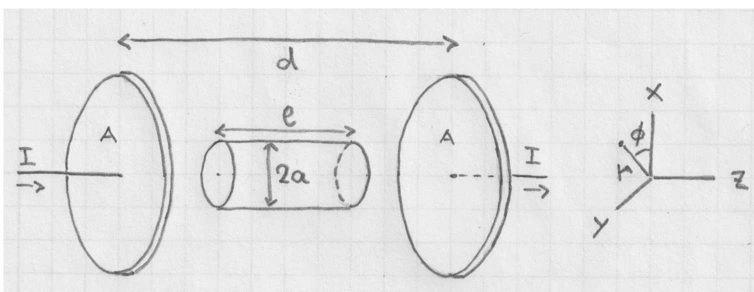
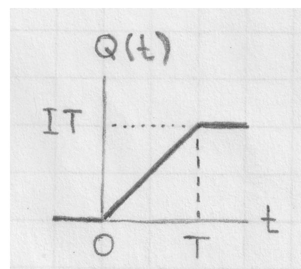
[lex142] Field energy between capacitor plates I

Consider a parallel-plate capacitor with plates of area A a distance d apart in the process of being a charged with a steady current I beginning at $t = 0$ and ending at $t = T$. Near its center, inside the cylinder of length l and radius a, a uniform electric field $\mathbf{E}(t)$ is being built up in the process.
(a) Express $\mathbf{E}(t)$ during the charging process in cylindrical coordinates as a function of I, C, d, t where $C = \epsilon_0 A/d$ is the capacitance.
(b) Calculate the electric-field energy U_E inside the cylinder as a function of I, T, C, d, a, l once the capacitor is fully charged.
(c) During the charging process, a steady but not uniform magnetic field $\mathbf{B}$ with a radial profiles is present as well inside the cylinder. Express $\mathbf{B}$ in cylindrical coordinates as a function of I, C, d, r.
(d) Determine the Poynting vector $\mathbf{S}(t)$ associated with the two fields during the charging process and infer from the result the direction by which field energy enters the cylinder.
(e) Calculate the flux $\Phi_S(t)$ of energy current density $\mathbf{S}(t)$ across the surface of the cylinder. Then integrate that quantity over the time of the charging process to recover U_E from part (b).
(f) Calculate the (time-independent) magnetic field energy U_B present inside the cylinder during the charging process. Express U_B as a function of a, l, I, C, d.
(g) Show that the ratio U_B/U_E of the magnetic field energy during charging and electric field energy after charging is a function of a/cT for the process described.
Note: Most expressions also contain factors of ϵ_0, μ_0, or $c = 1/\sqrt{\epsilon_0\mu_0}$.





Solution: