[lex141] Circularly polarized plane electromagnetic wave

Consider a plane electromagnetic wave characterized by an electric field of the form,

$$\mathbf{E}(\mathbf{x},t) = E_0\big[\sin(\mathbf{k}\cdot\mathbf{x}-\omega t)\,\hat{\mathbf{j}} + \cos(\mathbf{k}\cdot\mathbf{x}-\omega t)\,\hat{\mathbf{k}}\big], \quad \mathbf{k} = k_x\,\hat{\mathbf{i}} + k_y\,\hat{\mathbf{j}} + k_z\,\hat{\mathbf{k}}, \quad \mathbf{x} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}.$$

(a) Determine the associated expression for the magnetic field $\mathbf{B}(\mathbf{x},t)$ that makes the plane wave circularly polarized and traveling in the positive x-direction. Simplify the expression for $\mathbf{E}(\mathbf{x},t)$ in the process.
(b) Determine the Poynting vector $\mathbf{S}(\mathbf{x},t)$ and the energy density $u(\mathbf{x},t)$ of this wave. Simplify both expressions as much as possible.

Solution: