[lex140] Conduction current versus displacement current II
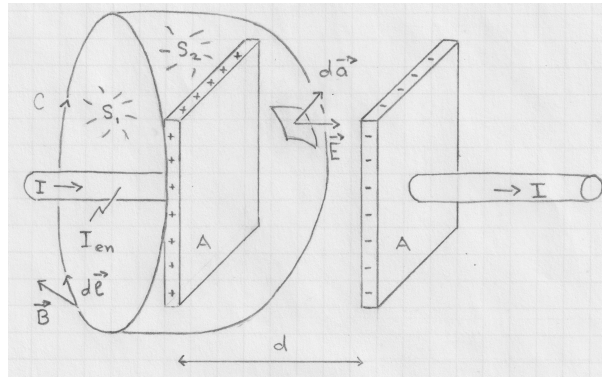
The setup is as in [lex82]: a parallel-plate capacitor with plates of area A a distance d apart is being charged by a conduction current I in on the left and out on the right via connecting wires. However, here the surrounding medium is both paramagnetic and dielectric. Ampère's law in integral form now reads

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{a}, \quad \mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H},$$

where $\mathbf{J}_f$ is the free current density, ϵ the permittivity and μ the permeability of the medium. The loop C is perimeter to surfaces S_1 and S_2. Show that the right-hand side of Ampère's law is identical for both surfaces, even though for surface S_1 there is only free current density and for surface S_2 only displacement current density.

Solution: