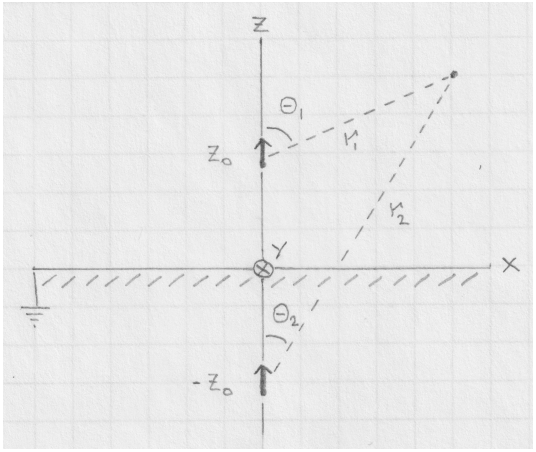
[lex14] Electric dipole near plane conducting surface I

When an electric dipole is placed near a plane conducting surface, its electric field initially penetrates the conductor. The mobile charge carriers then rearrange themselves toward electrostatic equilibrium such that the electric field is zero inside the conductor and directed perpendicular to the surface just outside the conductor.
Consider the case where the conductor fills the space at $z < 0$ and the electric dipole is oriented in z-direction, $\mathbf{p} = p_0\,\hat{\mathbf{k}}$, and placed on the z-axis at $z_0 > 0$.
(a) Use the method of images to calculate the radial profile of the surface charge density $\sigma(\bar{r})$, where $\bar{r} = \sqrt{x^2 + y^2}$. Plot that profile using scales that produce a universal curve.
(b) Calculate the induced charge q_{ind} via integration.
(c) Find the radius $\bar{r}_0$ at which the surface charge density σ changes sign and the (perpendicular) electric field $\mathbf{E}$ switches direction.
(d) Explain the decay law at large $\bar{r}$ of the induced surface charge density and electric field at near the surface.



Solution: