[lex137] Conducting ring moving into region of magnetic field I

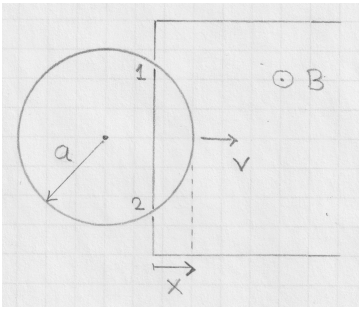
Consider a conducting ring of radius a and resistance R being moved at constant velocity v into a region of uniform magnetic field B directed as shown. The segment of the ring inside the region of field produces a motional EMF,

$$\mathcal{E} = \int_1^2 d\mathbf{l} \cdot \mathbf{v} \times \mathbf{B}, \tag{1}$$

where the integration is along the arc between points 1 and 2. This EMF causes a current I to flow around the ring.
(a) Calculate the distance $l(x)$ between points 1 and 2 (straight line) explicitly.
(b) Show that the induced EMF derived from (1) is $\mathcal{E}(x) = vBl(x)$.
(c) Plot $\mathcal{E}$ versus x appropriately scaled.
(d) Calculate the magnitude and direction (left/right) of the magnetic force $F(x)$ which the current-carrying ring experiences as it enters the region of magnetic field.
(e) Plot $F(x)$ versus x appropriately scaled.
(f) Calculate the mechanical work,

$$W = \int_0^{2a} dx F(x),$$

needed to move the ring into the region of field at constant speed v as a function of B, v, a, R.
(g) Integrate the power $P = \mathcal{E}^2/R$ dissipated in the ring to calculate the energy converted during entry, which should come out to be W.



Solution: