[lex135] Scalar and vector potentials in the Coulomb gauge

The Coulomb gauge condition, $\nabla \cdot \mathbf{A} = 0$, reduces Maxwell's equations for the scalar and vector potentials $\Phi(\mathbf{x})$ and $\mathbf{A}(\mathbf{x})$ as follows:

$$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0}, \quad \nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} + \frac{1}{c^2}\nabla \frac{\partial \Phi}{\partial t}.$$

Show that they can be decoupled into

$$\frac{1}{c^2}\nabla \frac{\partial \Phi}{\partial t} = \mu_0 \mathbf{J}_\mathrm{L}, \quad \nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}_\mathrm{T},$$

where

$$\mathbf{J} = \mathbf{J}_\mathrm{L} + \mathbf{J}_\mathrm{T}, \quad \nabla \times \mathbf{J}_\mathrm{L} = 0, \quad \nabla \cdot \mathbf{J}_\mathrm{T} = 0,$$

are the (irrotational) longitudinal and (solenoidal) transverse parts of the current density for which explicit expressions are derived in [lex134].

Solution: