[lex131] Magnetic dipole interaction II

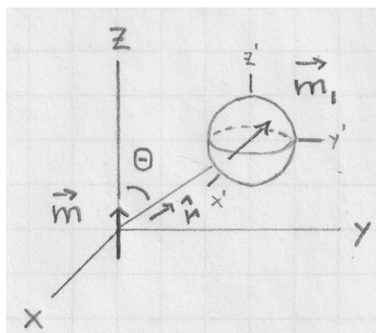
A magnetic dipole $\mathbf{m}$ at the origin of the coordinate system generates the magnetic field [lex36],

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{3\hat{\mathbf{r}}\,(\mathbf{m}\cdot\hat{\mathbf{r}}) - \mathbf{m}}{r^3}, \quad r = |\mathbf{x}|, \quad \hat{\mathbf{r}} = \frac{\mathbf{x}}{r},$$

When a second magnetic dipole $\mathbf{m}_1$ is placed into this field at position $\mathbf{x}$, the interaction potential energy is $U = -\mathbf{m}_1 \cdot \mathbf{B}(\mathbf{x})$. Consider the case where $\mathbf{m}$ is oriented in z-direction while $\mathbf{m}_1$ is placed in the yz-plane as in [lex131]. Here we allow $\mathbf{m}$ to be oriented out of the yz-plane and specify its orientation by the two angles ψ, ϕ via $x' = \sin\psi\cos\phi$, $y' = \sin\psi\sin\phi$, $z' = \cos\psi$.
(a) Express the scaled interaction energy $\bar{U}(\theta,\psi,\phi)$ constructed from

$$U = \frac{\mu_0}{4\pi}\frac{mm_1}{r^3}\,\bar{U}(\theta,\psi,\phi),$$

as a function of the angles $0 \leq \theta \leq \pi$, $-\pi \leq \psi \leq \pi$ and $-\pi \leq \phi \leq \pi$.
(b) Identify all angular positions θ for which the energetically most favorable orientation of $\mathbf{m}_1$ is independent of the azimuthal angle ϕ.
(c) Show that for all other angular positions θ the energetically most favorable orientation of $\mathbf{m}_1$ is in the plane spanned by vectors $\mathbf{m}$ and $\hat{\mathbf{r}}$.



Solution: