[lex130] Magnetic dipole interaction I

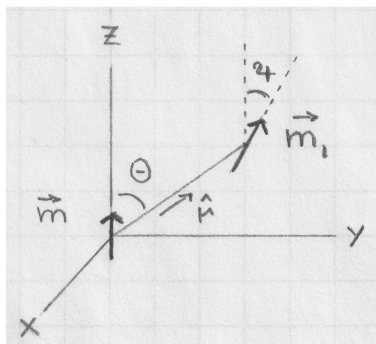
A magnetic dipole moment $\mathbf{m}$ at the origin of the coordinate system generates the magnetic field,

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{3\hat{\mathbf{r}}\,(\mathbf{m}\cdot\hat{\mathbf{r}}) - \mathbf{m}}{r^3}, \qquad r = |\mathbf{x}|, \quad \hat{\mathbf{r}} = \frac{\mathbf{x}}{r}.$$

When a second magnetic dipole moment $\mathbf{m}_1$ is placed into this field at position $\mathbf{x}$, the interaction potential energy is $U = -\mathbf{m}_1 \cdot \mathbf{B}(\mathbf{x})$. Consider the case where $\mathbf{m}$ is oriented in z-direction, while $\mathbf{m}_1$ is placed in the yz-plane and oriented at angle ψ away from the z-axis in the yz-plane as shown.
(a) Express the scaled interaction energy $\bar{U}(\theta,\psi)$ constructed from

$$U = \frac{\mu_0}{4\pi}\frac{mm_1}{r^3}\,\bar{U}(\theta,\psi), \qquad m = |\mathbf{m}|, \quad m_1 = |\mathbf{m}_1|,$$

as a function of the angles $0 \leq \theta \leq \pi$ and $-\pi \leq \psi \leq \pi$.
(b) Find the energetically most favorable orientation ψ of $\mathbf{m}_1$ at angular positions (i) $\theta = 0$, (ii) $\theta = \pi/2$, and (iii) $\theta = \pi/4$.
(c) Find the energetically most favorable angular position θ for a magnetic moment $\mathbf{m}_1$ oriented at angle $\psi = \pi/2$.



Solution: