[lex13] Point charge near plane conducting surface

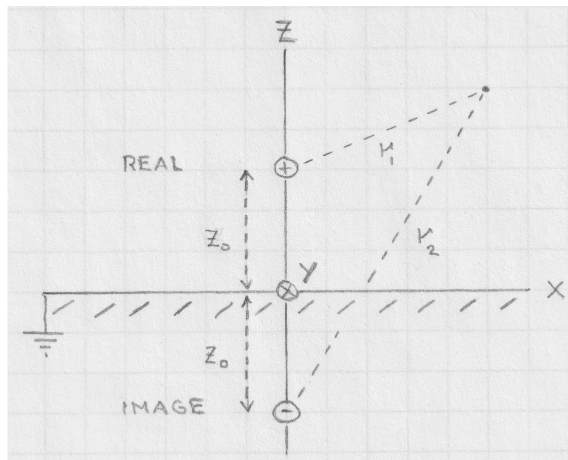
A positively charged particle positioned near a plane conducting surface attracts negative mobile charge carriers (electrons). Electrostatic equilibrium is restored when the electric field just outside the conductor is perpendicular to its surface. Assume the conducting surface to be in the xy-plane and the particle with charge $+q$ on the z-axis at $z_0 > 0$.

(a) Use the method of images to find the induced surface charge density $\sigma(x, y)$. Invoke the rotational symmetry about the z-axis to rewrite the charge density in the form $\sigma(x, y) = \bar{\sigma}(r)$, where $r = \sqrt{x^2 + y^2}$.

(b) Calculate the total induced charge q_{ind} both as an integral of $\sigma(x, y)$ and an integral of $\bar{\sigma}(r)$.

(c) With proper scaling the result for $\bar{\sigma}(r)$ can be graphically represented as one universal curve: for this purpose plot $z_0^2\bar{\sigma}/q$ versus r/z_0.

(d) The induced charge density, which is proportional to the electric field at the surface of the conductor decays faster, $\sim r^{-3}$, at large r, than the electric field of the point charge q or its image does, namely $\sim r^{-2}$. Explain this observation.



Solution: