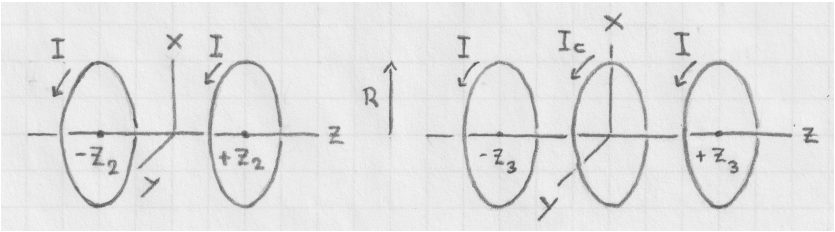
[lex129] Helmholtz coil and beyond

The purpose of the Helmholtz coil is to approximate a uniform magnetic field on the axis of two coaxial circular currents I with radius R, centered at $z = \pm z_2$ as shown on the left. The magnetic field field $B_2(z)$ on the axis at $z = 0$ is in z-direction and has vanishing odd derivatives for reasons of symmetry. Uniformity is optimized by enforcing a vanishing second derivative. This can be accomplished by adjusting the value of z_2.
(a) Find the optimal value of z_2/R that makes the the second derivative of $B_2(z)$ vanish at $z = 0$.
(b) Plot $B_2(z)R/\mu_0 I$ versus z/R for three values of z_2: the optimal value, one value a little higher and one value a little lower. Choose the latter two such as to visually demonstrate the type of deviations that ensue.

An improved uniformity of magnetic field can be accomplished by three circular currents of equal radius R and equal current directions positioned coaxially as shown on the right. The two outer loops are centered at $z = \pm z_3$ and have currents I. The loop at $z = 0$ has current I_c.
(c) Find the optimal values of z_3/R and I_c/I that make the second and fourth derivatives of $B_3(z)$ on the axis vanish at $z = 0$.
(d) Design a comparative plot $B_2(z)R/\mu_0 I$ for optimized z_2 and $B_3(z)R/\mu_0 I$ for optimized z_3, I_c versus z/R to demonstrate the improvement.



Solution: