

[lex12] Spherical capacitor

The device property *capacitance* for two oppositely charged conductors is defined as $C = Q/V$, where Q is the magnitude of charge on each capacitor and $V \doteq \Phi_2 - \Phi_1$ the potential difference between the two conductors [ln6]. Consider the case of two concentric conducting spheres of length L , one grounded and the other at potential Φ_0 . The inner sphere has radius a and the surrounding shell has inner radius b as shown. Calculate the result,

$$C = \frac{4\pi\epsilon_0 ab}{b - a},$$

for this geometry by going through the following steps: (i) coordinate system adapted to the spherical symmetry; (ii) Laplace equation for electric potential; (iii) boundary conditions; (iv) solution of Laplace equation; (v) electric field; (vi) charge density on relevant surfaces; (vii) charge on each conductor; (viii) voltage between conductors; (ix) capacitance.

Solution:

