[lex119] Larmor formula for radiation of accelerated charged particle

Consider a particle with charge q in motion with instantaneous acceleration $\mathbf{a}$. The magnetic and electric fields in the radiation zone a distance R from the source are

$$\mathbf{B}(\mathbf{x},t)_{\text{rad}} = -\frac{\mu_0 q}{4\pi c}\frac{\hat{\mathbf{R}}(t_r)\times\mathbf{a}(t_r)}{R(t_r)}, \quad \mathbf{E}(\mathbf{x},t)_{\text{rad}} = c\,\mathbf{B}(\mathbf{x},t)_{\text{rad}}\times\hat{\mathbf{R}}(t_r),$$

respectively, as established in [lln19].
(a) Show that the Poynting vector can be rendered as follows:

$$\mathbf{S}(\mathbf{x},t)_{\text{rad}} = \frac{\mu_0 q^2}{(4\pi)^2 R^2 c}\left[a^2 - \left(\mathbf{a}\cdot\hat{\mathbf{R}}\right)^2\right]\hat{\mathbf{R}}.$$

(b) Show that the instantaneous power radiated through a closed surface around the source is the Larmor result,

$$P(t) = \frac{1}{4\pi\epsilon_0}\frac{2q^2a^2(t_r)}{3c^3}.$$

Solution: