

[lex118] Radiation fields and Poynting vector for half-wave linear antenna

Given the vector potential in the radiation zone,

$$\mathbf{A}(\mathbf{x}, t)_{\text{rad}} = -\frac{\mu_0 I_0}{2\pi k r} \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin^2\theta} \sin(kr - \omega t) \hat{\mathbf{z}},$$

for a half-wave linear antenna as derived in [lln19], calculate the the radiation magnetic field from $\mathbf{B} = \nabla \times \mathbf{A}$, the radiation electric field from $\mathbf{E} = c\mathbf{B} \times \hat{\mathbf{r}}$, and the Poynting vector in the radiation zone from $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$.

Hint: Use spherical coordinates and neglect all field terms that decay faster than $\sim r^{-1}$.

Solution: