

[lex117] Antenna resistance of Hertzian dipole

The quantification of power characteristics for an antenna employs an analogy with common resistors in ac circuits, which relates the average power dissipated to the current: $P_{\text{av}} = RI_{\text{rms}}^2$. The current amplitude in a Hertzian dipole is $I_0 = \omega Q_0$. The amplitude of the electric dipole moment is $p_0 = Q_0 d$, where d is a characteristic length of the dipole. Show that the antenna resistance is

$$R = \frac{2\pi}{3} \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{d}{\lambda}\right)^2,$$

where λ is the wavelength of the radiation. Note that the antenna resistance only depends on the ratio of two characteristic lengths in units of $\sqrt{\mu_0/\epsilon_0} \simeq 377\Omega$, known as impedance of free space.

Solution: