[lex115] Asymptotic scalar potential of electric dipole radiation

Given the asymptotic vector potential,

$$\Rightarrow \ \mathbf{A}(\mathbf{x},t)_{\rm as} = \frac{\mu_0}{4\pi r}\left.\frac{d\mathbf{p}}{dt}\right|_{t_r}, \quad t_r \doteq t - \frac{r}{c},$$

generated by a time-dependent electric dipole $\mathbf{p}(t)$ as worked out in [lln19], use the Lorenz gauge condition, $\partial\Phi/\partial t = -c^2\nabla\cdot\mathbf{A}$, to show that the asymptotic scalar potential in the radiation zone becomes

$$\Phi(\mathbf{x},t)_{\rm as} = \frac{\mu_0 c}{4\pi r}\,\hat{\mathbf{r}}\cdot\left.\frac{d\mathbf{p}}{dt}\right|_{t_r}.$$

Solution: