[lex113] Electric dipole moment from charge density or current density

The electric dipole moment of a charge distribution (assumed to be confiend to a compact region of space) is defined by the integral expression,

$$\mathbf{p}(t) = \int d^3x\, \mathbf{x}\rho(\mathbf{x}, t).$$

Charge conservation relates the charge density to the current density by the continuity equation,

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0.$$

Show that the following relation between the electric dipole moment and the current density follows:

$$\frac{d}{dt}\,\mathbf{p}(t) = \int d^3x\, \mathbf{J}(\mathbf{x}, t).$$

Hint: Integrate $d\mathbf{a} \cdot x_i\mathbf{J} = 0$, $i = 1, 2, 3$ over closed surfaces surrounding the region of nonzero charge density, then apply Gauss's theorem.

Solution: