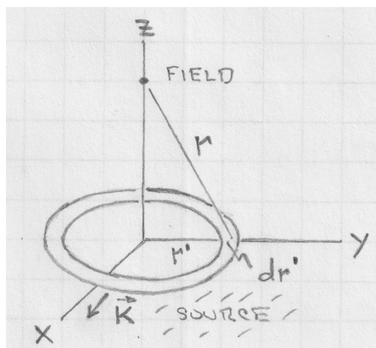
[lex112] Planar surface current abruptly established

A surface current with current density, $\mathbf{J}(\mathbf{x},t) = K_0\,\theta(t)\,\delta(z)\,\hat{\mathbf{i}}$, in the xy-plane and moving in x-direction is abruptly established to the constant value K_0 at time $t = 0$. No charge density is involved: $\rho \equiv 0$.
(b) Start from the expression,

$$\mathbf{A}(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int d^3x'\,\frac{\mathbf{J}(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|/c)}{|\mathbf{x}-\mathbf{x}'|},$$

developed in [lln19] for the vector potential, to calculate $\mathbf{A}(z,t)$ explicitly for the situation at hand.
(b) Infer expressions for the magnetic field $\mathbf{B}(z,t)$ and the electric field $\mathbf{E}(z,t)$. Calculate the Poynting vector $\mathbf{S}(z,t)$. Which of the three vectors change direction between $z > 0$ and $z < 0$.
(c) Show that the fields $\mathbf{E}$ and $\mathbf{B}$ (at $z > 0$) satisfy Faraday's law and Ampère's law.
(d) Explain why radiation near the source persists for all $t > 0$ even though charge acceleration happens only at $t = 0$.
Hint: Use polar coordinates as shown to carry out the integral.



Solution: