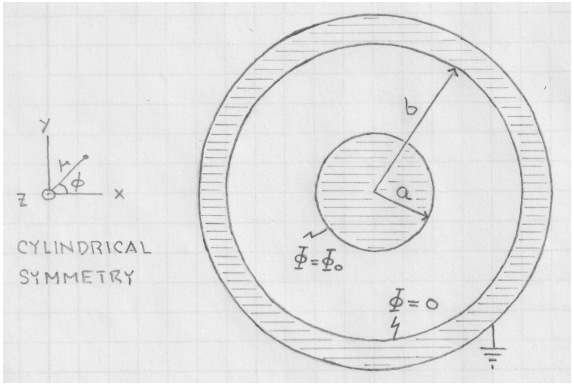
[lex11] Cylindrical capacitor

The device property *capacitance* for two oppositely charged conductors is defined as $C = Q/V$, where Q is the magnitude of charge on each capacitor and $V \doteq \Phi_2 - \Phi_1$ the potential difference between the two conductors [lln6]. Consider the case of two coaxial conducting cylinders of length L, one grounded and the other at potential Φ_0. The inner cylinder has radius a and the surrounding shell has inner radius b as shown. Edge effects are negligible if $L \gg a, b$, as assumed to be the case here. Calculate the result,

$$C = \frac{2\pi\epsilon_0 L}{\ln(b/a)},$$

for this geometry by going through the following steps: (i) coordinate system adapted to the cylindrical symmetry; (ii) Laplace equation for electric potential; (iii) boundary conditions; (iv) solution of Laplace equation; (v) electric field; (vi) charge density on relevant surfaces; (vii) charge on each conductor; (viii) voltage between conductors; (ix) capacitance.



Solution: