[lex109] TEM mode in coaxial cable I: electric and magnetic fields

The ansatz,

$$\mathbf{E}(\mathbf{x},t) = E_r(r)e^{\imath(kz-\omega t)}\,\hat{\mathbf{r}}, \qquad \mathbf{B}(\mathbf{x},t) = B_\phi(r)e^{\imath(kz-\omega t)}\,\hat{\boldsymbol{\phi}},$$

for the electric and magnetic fields of a TEM wave in a coaxial cable with cross-sectional specifications as shown, is inspired by symmetry and the boundary conditions $\mathbf{E}_\parallel = \mathbf{B}_\perp = 0$.
(a) Show that the ansatz solves Maxwell's equations if

$$E_r(r) = vB_\phi(r) = \frac{C_0}{r}, \quad v = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu_0}}.$$

(b) Show that the amplitude of the potential difference between the two conductors is

$$\Phi_0 = C_0 \ln(b/a).$$

Solution: