[lex103] Dispersion and absorption in a dielectric

Atomic polarizability is shown in [lln17] to elicit a response to a the oscillating electric field of an incident electromagnetic wave in the form of a frequency-dependent permittivity,

$$\epsilon(\omega) = \epsilon_0 + \nu\alpha(\omega), \quad \alpha = \frac{e^2}{K - m\omega^2 - \imath\omega\gamma},$$

where ν is the density of atoms. Show that in the limit of weak polarizability, $\nu\alpha \ll \epsilon_0$, the index of refraction and the adsorption length, as inferred from dispersion relation,

$$\kappa(\omega) = \omega\sqrt{\epsilon(\omega)\mu_0} \doteq \kappa_1(\omega) + \imath\kappa_2(\omega),$$

are represented by the following expressions:

$$n = \frac{c\kappa_1}{\omega} = 1 + \frac{\nu e^2}{2\epsilon_0 m\omega_0^2}\frac{\omega_0^2(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma/m)^2}, \quad \omega_0 \doteq \sqrt{\frac{K}{m}}$$
$$d^{-1} = 2\kappa_2 = \frac{\nu e^2}{\epsilon_0 mc}\frac{\omega^2\gamma/m}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma/m)^2}.$$

Solution: