[lex101] Reflection of electromagnetic wave from conductor

Consider a plane wave of the form $\mathbf{E}(\mathbf{x},t) = \mathbf{E}_0 e^{\imath(\kappa x - \omega t)}$ with $\mathbf{E}_0 \perp \hat{\mathbf{i}}$ being reflected from a conductor with permittivity ϵ, permeability μ, and conductivity σ. The surface is in the yz-plane. The goal is to derive an expression for the reflectivity for a good conductor (a material with $\sigma \gg \epsilon\omega$). The complex wave number of the wave in a good conductor is [lln17] [lex100]

$$\kappa = \kappa_1 + \imath\kappa_2, \quad \kappa_1 \simeq \kappa_2 \simeq \sqrt{\frac{\mu\sigma\omega}{2}}.$$

Starting from the expression [lln17],

$$\frac{E_0''}{E_0} = \frac{\mu_2 n_1 - \mu_1 n_2}{\mu_2 n_1 + \mu_1 n_2},$$

for the amplitude ratio between reflected and incident wave in a case of normal reflection, use the specifications $\mu_1 = \mu_2 = \mu_0$, $n_1 = 1$, $n_2 = c/v_2 = c\kappa/\omega$ to derive the following expression for the reflectivity of a good conductor:

$$R = 1 - \sqrt{\frac{8\omega\epsilon_0}{\sigma}}.$$

Solution: