[lex100] Electromagnetic wave in a conductor

In a metal (uncharged material with permittivity ϵ, permeability μ, and conductivity σ), Maxwell's equations read [lln17],

$$\nabla \cdot \mathbf{E} = 0, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu\sigma\mathbf{E} + \mu\epsilon\frac{\partial \mathbf{E}}{\partial t}.$$

(a) Show that the electric an magnetic fields then satisy wave equations amended by an attenuation term as follows:

$$\nabla^2\mathbf{E} = \mu\sigma\,\frac{\partial \mathbf{E}}{\partial t} + \mu\epsilon\,\frac{\partial^2 \mathbf{E}}{\partial t^2}, \qquad \nabla^2\mathbf{B} = \mu\sigma\,\frac{\partial \mathbf{B}}{\partial t} + \mu\epsilon\,\frac{\partial^2 \mathbf{B}}{\partial t^2}.$$

(b) Show that a plane-wave solution of the form $\mathbf{E}(\mathbf{x}, t) = \mathbf{E}_0 e^{\imath(\kappa x - \omega t)}$ with $\mathbf{E}_0 \perp \hat{\mathbf{i}}$, is a solution for the complex wave number, $\kappa = \kappa_1 + \imath\kappa_2$, where

$$\kappa_1 = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} + 1\right]^{1/2}, \quad \kappa_2 = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1\right]^{1/2}.$$

Solution: