[lex10] Parallel-plate capacitor

The device property *capacitance* for two oppositely charged conductors is defined as $C = Q/V$, where Q is the magnitude of charge on each capacitor and $V \doteq \Phi_2 - \Phi_1$ the potential difference between the two conductors [lln6]. Consider the case of two rectangular and parallel conducting plates of area $A = L_1 L_2$. One plate is grounded and the other at potential Φ_0. The two plates are separated a distance d from each other as shown. Edge effects are negligible if $L_1, L_2 \gg d$, as assumed to be the case here. Calculate the result,

$$C = \frac{\epsilon_0 A}{d},$$

for this geometry by going through the following steps: (i) coordinate system adapted to the planar symmetry; (ii) Laplace equation for electric potential; (iii) boundary conditions; (iv) solution of Laplace equation; (v) electric field; (vi) charge density on relevant surfaces; (vii) charge on each conductor; (viii) voltage between conductors; (ix) capacitance.

Solution: