Quantum Optics II [lln26]

Here we continue the discussion of quantum optics from [lln24] on an introductory level, in preparation of a presentation at a more advanced level.

Photon bunching and antibunching:

An alternative characterization of light as a stream of photons is based on the time correlation of the intensity, measurable by an intensity interferometer (a device developed by astronomers).

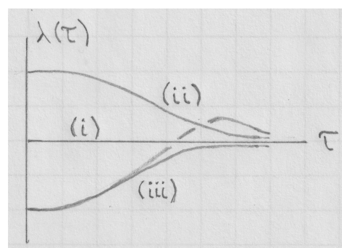
Second-order correlation function [lam2]:

$$g^{(2)}(\tau) \doteq \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle}, \quad \lambda(\tau) \doteq g^{(2)}(\tau) - 1.$$

The brackets include long-time averages. Under conditions of stationarity, we can use $\langle I(t)\rangle = \langle I(t+\tau)\rangle$.

The stream of photons is divided into regimes according to short-time intensity correlations:

(i) $\lambda(\tau) \equiv 0$: coherent light,
(ii) $\lambda(0) > \lambda(\tau)$: bunched light,
(iii) $\lambda(0) < \lambda(\tau)$: antibunched light.



Bunching is associated with sources of intensity fluctuations and with the sources of line broadening discussed in [lln24]. Antibunching is a quantum effect.

Light modeled as harmonic oscillations:

Consider a harmonic oscillator in one dimension.

– Hamiltonian: $\mathcal{H} = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2 x^2$.

– Canonical equations: $\dot{x} = \frac{\partial \mathcal{H}}{\partial p_x} = \frac{p_x}{m}, \quad \dot{p}_x = -\frac{\partial \mathcal{H}}{\partial x} = -m\omega^2 x$.

– Lagrange equation of motion: $\ddot{x} = -\omega^2 x$.

– Solutions: $x(t) = x_0 \sin(\omega t + \phi), \quad p_x(t) = \overbrace{m\omega x_0}^{p_0}\cos(\omega t + \phi)$.

– Scaled variables: $q(t) \doteq \sqrt{m}x(t), \quad p(t) \doteq \frac{p_x(t)}{\sqrt{m}}$.

– Scaled Hamiltonian: $\mathcal{H} = \frac{1}{2}\big(p^2 + \omega^2 q^2\big)$.

Application to a linearly polarized standing electromagnetic wave of wavelength λ in a cavity of width L.

– Electric field: $E_x(z,t) = E_0 \sin(kz)\sin(\omega t), \quad k = \frac{2\pi}{\lambda}, \quad \omega = kc.$

– Boundary conditions: $\sin(kL) = 0 \quad \Rightarrow\ k = n\pi/L, \quad n = 1, 2, \ldots$

– Ampère's law: $\nabla \times \mathbf{B} = \epsilon_0\mu_0 \frac{\partial \mathbf{E}}{\partial t}, \quad \big(\nabla \times \mathbf{B}\big)_x = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right).$

$$\Rightarrow \quad -\frac{\partial B_y}{\partial z} = \epsilon_0\mu_0 \frac{\partial E_x}{\partial t} = \epsilon_0\mu_0\omega E_0 \sin(kz)\cos(\omega t).$$

– Magnetic field: $B_y(z,t) = B_0 \cos(kz)\cos(\omega t), \quad B_0 = \frac{E_0}{c},\ c = \frac{1}{\sqrt{\epsilon_0\mu_0}}.$

– Energy density: $u = \frac{\epsilon_0}{2} E_0^2 \sin^2(kz)\sin^2(\omega t) + \frac{1}{2\mu_0} B_0^2 \cos^2(kz)\cos^2(\omega t).$

– Field energy: $U = \frac{V}{4}\left[\epsilon_0 E_0^2 \sin^2(\omega t) + \frac{1}{\mu_0} B_0^2 \cos^2(\omega t)\right].$

– Equivalent oscillator coordinates: $q(t) = \sqrt{\frac{\epsilon_0 V}{2\omega^2}}\, E_0 \sin(\omega t),$

$$p(t) = \sqrt{\frac{V}{2\mu_0}}\, B_0 \cos(\omega t) = \sqrt{\frac{\epsilon_0 V}{2}}\, E_0 \cos(\omega t).$$

– Equivalent field energy: $U = \mathcal{H} = \frac{1}{2}\big(p^2 + \omega^2 q^2\big)$.

The energy of the magnetic (electric) field in the standing wave is represented by the kinetic (potential) energy of the oscillator.

Phasor diagram and field quadratures:

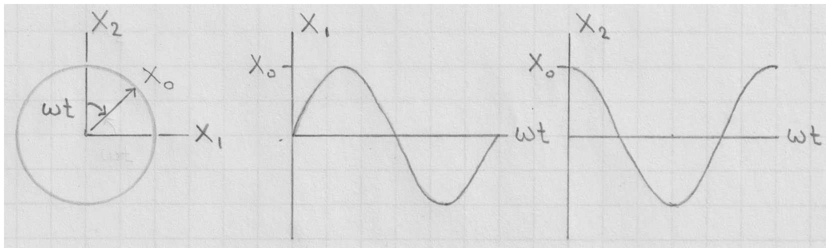
Consider the electric field of a standing wave with arbitrary phase.

$$\begin{aligned} E_x(z,t) &= E_0 \sin(kz)\sin(\omega t + \phi) \\ &= E_0 \sin(kz)\cos\phi\sin(\omega t) + E_0 \sin(kz)\sin\phi\cos(\omega t) \end{aligned}$$

Field quadratures are dimensionless and split the electric field into two parts out of phase by 90°. The rescaling is tailor-made for quantum optics:

$$X_1(t) = X_0 \sin(\omega t), \quad X_2(t) = X_0 \cos(\omega t); \quad X_0 = \sqrt{\frac{\epsilon_0 V}{4\hbar\omega}}\, E_0.$$

$$\Rightarrow\; E_x(z,t) = \sqrt{\frac{4\hbar\omega}{\epsilon_0 V}}\,\sin(kz)\big[\cos\phi\, X_1(t) + \sin\phi\, X_2(t)\big].$$



Mapping to harmonic oscillator and quantization:

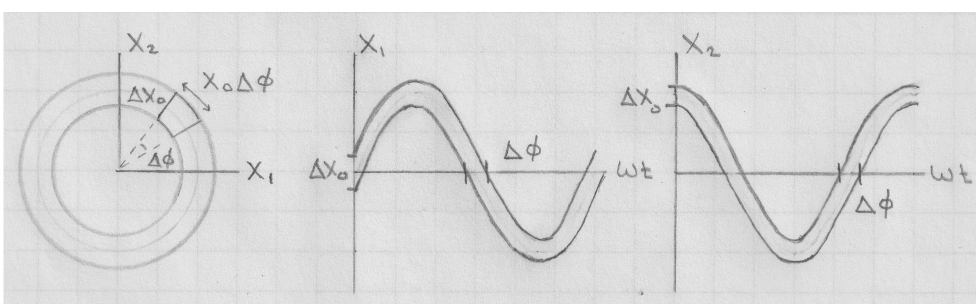
$$X_1(t) = \sqrt{\frac{\omega}{2\hbar}}q(t), \quad X_2(t) = \sqrt{\frac{1}{2\hbar\omega}}p(t),$$

$$\hbar\omega\big[X_1^2(t) + X_2^2(t)\big] = \hbar\omega X_0^2 = \frac{1}{4}\epsilon_0 E_0^2 V = \frac{1}{2}(p^2 + \omega^2 q^2) = \hbar\omega\left(\langle n\rangle + \frac{1}{2}\right).$$

Impact of quantum uncertainty:

$$\Delta x \Delta p_x \geq \frac{1}{2}\hbar \;\;\Rightarrow\;\; \Delta q \Delta p \geq \frac{1}{2}\hbar \;\;\Rightarrow\;\; \Delta X_1 \Delta X_2 = \sqrt{\frac{\omega}{2\hbar}}\sqrt{\frac{1}{2\hbar\omega}}\Delta q \Delta p \geq \frac{1}{4}.$$

Switch to polar coordinates: $(X_0 \Delta\phi)\Delta X_0 \geq \dfrac{1}{4}$.



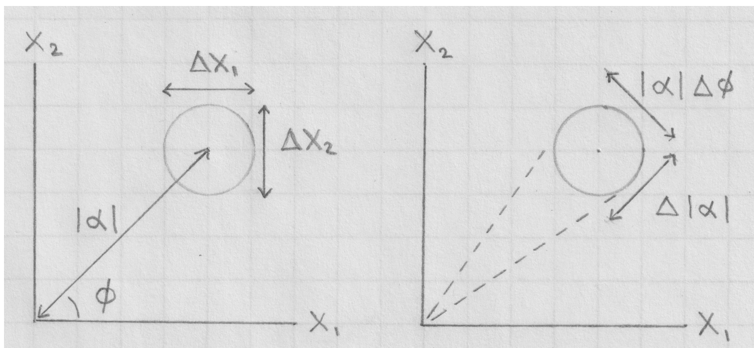
- ▷ ΔX_0 represents an uncertainty in amplitude and/or frequency.
- ▷ $\Delta\phi$ represents an uncertainty in phase.

Coherent states of light:

Coherent light is described quantum mechanically by coherent states. A more formal description follows later.

In the phasor diagram, a coherent state can be specified by polar coordinates:

$$|\alpha| = X_0 = \sqrt{X_1^2 + X_2^2}, \quad \phi = \arctan\frac{X_2}{X_1}.$$



The radial coordinate $|\alpha|$ is related to the average number of photons $\langle n\rangle$ in the coherent state (assuming $\langle n\rangle \gg 1$):

$$U_{\rm cl} = \frac{1}{4}\epsilon_0 E_0^2 V = \hbar\omega X_0^2 = \hbar\omega|\alpha|^2, \quad U_{\rm qu} = \hbar\omega\left(\langle n\rangle + \frac{1}{2}\right) \quad \Rightarrow\ \langle n\rangle = |\alpha|^2.$$

The variance $\langle\langle n^2\rangle\rangle$ of the photon count in a coherent state can be inferred from the uncertainty relation (assuming balanced minimum uncertainty):

$$\underbrace{\big(\Delta|\alpha|\big)}_{1/2}\underbrace{\big(|\alpha|\Delta\phi\big)}_{1/2} = \frac{1}{4} \quad \Rightarrow\ \Delta n = \Delta|\alpha|^2 = \underbrace{2\Delta|\alpha|}_{1}|\alpha| = |\alpha|.$$

$$\Rightarrow\ \langle\langle n^2\rangle\rangle \doteq \langle(\Delta n)^2\rangle = |\alpha|^2 = \langle n\rangle.$$

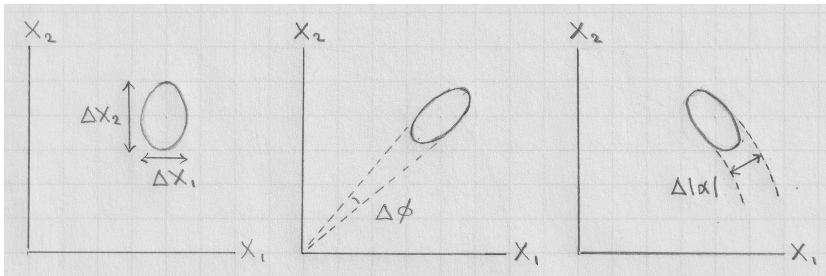
This relation between mean and variance confirms that coherent light is characterized by a photon stream with Poisson statistics.

What is the phase uncertainty in coherent light with $\langle n\rangle \gg 1$? The answer is provided by the minimum uncertainty (see diagram on the right):

$$|\alpha|\Delta\phi = \sqrt{\langle n\rangle}\Delta\phi = \frac{1}{2} \quad \Rightarrow\ \Delta\phi = \frac{1}{2\sqrt{\langle n\rangle}} \quad \Rightarrow\ \langle\langle\phi^2\rangle\rangle \doteq \langle(\Delta\phi)^2\rangle = \frac{1}{4\langle n\rangle}.$$

Squeezed states of light:

Squeezed light is characterized by unbalanced uncertainty as manifest in the phasor diagram.



– Coherent light: $\Delta X_1 = \Delta X_2 = \dfrac{1}{2} \overset{\langle n\rangle \gg 1}{\longrightarrow} \langle\langle n^2\rangle\rangle = \langle n\rangle, \quad \langle\langle \phi^2\rangle\rangle = \dfrac{1}{4\langle n\rangle}$.

– Quadrature-squeezed light: $\Delta X_1 < \dfrac{1}{2}, \quad \Delta X_2 > \dfrac{1}{2}$ or vice versa.

– Phase-squeezed light: $\langle\langle \phi^2\rangle\rangle < \dfrac{1}{4\langle n\rangle}, \quad \langle\langle n^2\rangle\rangle > \langle n\rangle$ (super-Poisson).

– Amplitude-squeezed light: $\langle\langle \phi^2\rangle\rangle > \dfrac{1}{4\langle n\rangle}, \quad \langle\langle n^2\rangle\rangle < \langle n\rangle$ (sub-Poisson).

Extreme amplitude squeezing, $\langle\langle n^2\rangle\rangle \to 0$, produces photon-number states, characterized by complete phase uncertainty.

Criteria for the classification of light:

We have introduced three ways of classifying light as streams of photons based on different criteria:

– *Statistics*: Poisson – super-Poisson – sub-Poisson.

– *Correlations*: coherent – bunched – anti-bunched.

– *Uncertainty*: balanced – phase-squeezed – amplitude-squezzed.

There is no one-to-one relationship between these criteria, but the criteria are not entirely independent.

Photonic number states:

The most elementary quantum theory of light is based on the quantum harmonic oscillator. Sketch of an operator solution [lam5].

Hamiltonian: $\mathcal{H} = \frac{1}{2m}\mathcal{P}^2 + \frac{1}{2}m\omega^2\mathcal{Q}^2$.

Ladder operators (raising/lowering, creation/annihilation):

$$a^\dagger = \frac{1}{\sqrt{2m\hbar\omega}}\big(m\omega\mathcal{Q} - \imath\mathcal{P}\big), \quad a = \frac{1}{\sqrt{2m\hbar\omega}}\big(m\omega\mathcal{Q} + \imath\mathcal{P}\big).$$

$$\Rightarrow \; \mathcal{P} = \imath\sqrt{\frac{m\hbar\omega}{2}}\big(a^\dagger - a\big), \quad \mathcal{Q} = \sqrt{\frac{\hbar}{2m\omega}}\big(a + a^\dagger\big).$$

Commutators: $[\mathcal{Q},\mathcal{P}] = \imath\hbar, \quad [a, a^\dagger] = 1, \quad [\mathcal{H}, a^\dagger] = \hbar\omega a^\dagger, \quad [\mathcal{H}, a] = -\hbar\omega a.$

Number representation of eigenstates:

– Number operator: $\mathcal{N} = a^\dagger a$.

– Hamiltonian: $\mathcal{H} = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$.

– Number states: $|n\rangle, \quad n = 0, 1, 2, \ldots$ with $\langle n|n'\rangle = \delta_{nn'}$.

– Action of ladder and number operators:

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \quad a|n\rangle = \sqrt{n}|n-1\rangle \quad \Rightarrow \; a^\dagger a|n\rangle = n|n\rangle.$$

– Energy levels: $E_n = \hbar\omega\left(\langle n|a^\dagger a|n\rangle + \frac{1}{2}\right) = \hbar\omega\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots$

– Generation of number states from ground state: $|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$.

– Quadrature field operators:

$$X_1 = \sqrt{\frac{m\omega}{2\hbar}}\,\mathcal{Q} = \frac{1}{2}\big(a^\dagger + a\big), \quad X_2 = \sqrt{\frac{1}{2\hbar\omega m}}\,\mathcal{P} = \frac{1}{2}\imath\big(a^\dagger - a\big).$$

– Relation to number operator: $X_1^2 + X_2^2 = a^\dagger a + \frac{1}{2}$.

Photonic number states are eigenstates of the quantum harmonic oscillator. The number $\langle n\rangle$ of photons is not fluctuating, implying $\langle\langle n^2\rangle\rangle = 0$.

These attributes represent extreme sub-Poisson statistics and extreme amplitude squeezing in the phasor diagram.

Photonic coherent states:

Coherent states are a normalized but non-orthogonal set of states of the same quantum harmonic oscillator.

They depend an a complex-valued parameter α and are related to the number states as follows:

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

The reasoning behind this construction will be elucidated later.

Coherent states have balanced minimum uncertainties in the position and momentum operators and are eigenstates of ladder operators [lex154]:

$$a|\alpha\rangle = \alpha|\alpha\rangle, \quad \langle\alpha|a^\dagger = \langle\alpha|\alpha^*.$$

The photon statistics of a coherent state follows directly [lex154]:

– Mean and variance: $\langle n\rangle = |\alpha|^2, \quad \langle\langle n^2\rangle\rangle = \langle n\rangle$.

– Distribution: $P(n) \doteq |\langle n|\alpha\rangle|^2 = \dfrac{\langle n\rangle^n}{n!}\, e^{-\langle n\rangle}$ (Poisson).

The statistics of coherent states of the quantum harmonic oscillator are consistent with coherent light identified earlier in the phasor diagram.

Coherent light is unsqueezed and has balanced minimum uncertainties in the quadrature fields.

Coherent states are the most localized quantum state in a phase-space representation (here the plane of x, p). They can by generated from the ground state by a translation to an arbitrary point in phase space.

– Point in phase plane: x, p.

– Parameter of coherent state: $\alpha = \sqrt{\dfrac{\omega m}{2\hbar}}\, x + \imath\sqrt{\dfrac{1}{2\hbar\omega m}}\, p$.

– Displacement operator [lex155]: $D(\alpha) = e^{\imath(p\mathcal{Q} - x\mathcal{P})/\hbar} = e^{\alpha a^\dagger - \alpha^* a}$.

– Coherent state from ground state: $D(\alpha)|0\rangle = e^{\alpha a^\dagger - \alpha^* a}|0\rangle = |\alpha\rangle$.

Time evolution from Schrödinger equation: $\mathcal{H}|\psi\rangle = \imath\hbar \dfrac{\partial}{\partial t}|\psi\rangle$.

– Stationary states: $|n(t)\rangle = |n\rangle e^{-\imath\omega_n t}, \quad \omega_n = \big(n + \frac{1}{2}\big)\omega$.

– Coherent state: $|\alpha(t)\rangle = \left[e^{-|\alpha_0|^2/2} \displaystyle\sum_{n=0}^{\infty} \frac{[\alpha_0 e^{-\imath\omega t}]^n}{\sqrt{n!}}\right] |n\rangle e^{-\imath\omega t/2}$.

Photonic thermal states:

Unlike number states and coherent states, which are pure quantum states, thermal states are mixed quantum states.

Only pure quantum states can be represented as state vectors, but all three kinds of photonic states can be represented by density operators [lam1].

– Number state: $\rho_\mathrm{N} = |n\rangle\langle n|$.

$$\mathrm{Tr}[\rho_\mathrm{N}] = \sum_{m=0}^{\infty} \langle m|n\rangle\langle n|m\rangle = 1.$$

– Coherent state: $\rho_\mathrm{C} = |\alpha\rangle\langle\alpha| = e^{-|\alpha|^2}\sum_{nm}\frac{(\alpha^*)^m\alpha^n}{\sqrt{m!n!}}|n\rangle\langle m|$.

$$\mathrm{Tr}[\rho_\mathrm{C}] = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!} = 1.$$

– Thermal state: $\rho_\mathrm{T} = \sum_{n=0}^{\infty} P_n|n\rangle\langle n|, \quad P_n = (1-e^{-\beta\hbar\omega})\, e^{-\beta n\hbar\omega}$.

$$\mathrm{Tr}[\rho_\mathrm{T}] = \sum_{n=0}^{\infty} P_n = (1-e^{-\beta\hbar\omega})\sum_{n=0}^{\infty}(e^{-\beta\hbar\omega})^n = 1.$$

Incoherent light more generally is represented by mixed quantum states whose density operator is not diagonal in the basis of number states.

Note that for all three types of photonic states we have used the number states $|n\rangle$ of the quantum harmonic oscillator as basis states.

In our next task it is the coherent states $|\alpha\rangle$ that are being employed for all three types of photonic states.

Phase-space representations of photonic states:

From the overlap $\langle\gamma|\psi\rangle$ of any pure quantum state $|\psi\rangle$ with the coherent states $|\gamma\rangle$ it is possible to construct a probability distribution in phase space.

The coherent state to be used for that purpose is centered at point (x, p) in phase space:

$$|x,p\rangle = e^{-|\gamma|^2/2}\sum_{n=0}^{\infty}\frac{\gamma^n}{\sqrt{n!}}|n\rangle, \quad \gamma = \sqrt{\frac{\omega m}{2\hbar}}\,x + \imath\sqrt{\frac{1}{2\hbar\omega m}}\,p = \bar{x} + \imath\bar{p}.$$

Husimi distribution for a pure quantum state $|\psi\rangle$ in general:

$$W_\psi(x,p) \doteq \frac{1}{\pi}|\langle x,p|\psi\rangle|^2.$$

The Husimi distribution thus defined satisfies the requirements of a probability distribution. It is non-negative everywhere and it is normalized. The factor $1/\pi$ compensates for the fact that coherent states are an overcomplete set as is evident in [lex185].

Note the difference from the absolute square of the wave function, $|\psi(x)|^2$, which yields a probability distribution in position space [lam5].

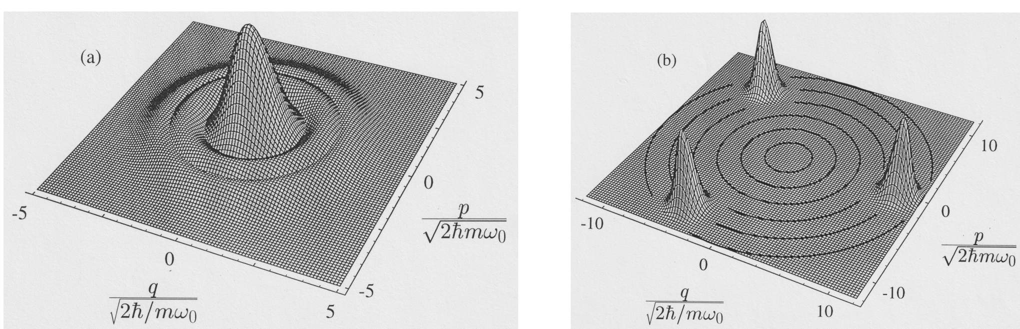
Application to the (stationary) number state $|n\rangle$:

$$W_n(x,p) = \frac{1}{\pi}|\langle x,p|n\rangle|^2 = \frac{1}{\pi}e^{-|\gamma|^2}\frac{|\gamma|^{2n}}{n!}, \quad |\gamma|^2 = \bar{x}^2 + \bar{p}^2.$$

Application to the coherent state centered at $\alpha = \bar{x}_0 + \imath\bar{p}_0$:

$$W_\alpha(x,p) = \frac{1}{\pi}|\langle x,p|\alpha\rangle|^2 = \frac{1}{\pi}e^{-(|\beta|^2+|\alpha|^2)}\sum_{n=0}^{\infty}\frac{(\gamma^*\alpha+\gamma\alpha^*)^n}{n!}$$
$$= \frac{1}{\pi}e^{-(|\gamma|^2+|\alpha|^2)}e^{\gamma^*\alpha+\gamma\alpha^*} = \frac{1}{\pi}e^{-|\gamma-\alpha|^2}.$$

Time-dependence of coherent state: set $\alpha = \alpha_0 e^{-\imath\omega t}$.



[images from Regez et al. 1996]

Left: Husimi distribution of eigenstates (here for $n = 0, 5, 9$) are time-independent and have rotational symmetry in the scaled phase plane $\bar{x}, \bar{p}$.

Right: Husimi distributions of coherent states are time-dependent. One is shown for three successive instants that span one period $2\pi/\omega$. The peak moves clockwise like a classical phase point would.

A natural generalization of Husimi distributions to mixed (M) quantum states starts from the density operator in the form

$$\rho_{\mathrm{M}} = \sum_\psi P_\psi |\psi\rangle\langle\psi|, \quad \sum_\psi P_\psi = 1,$$

as introduced in [lam1], and continues with the construction,

$$W_{\mathrm{M}}(x,p) = \frac{1}{\pi}\sum_\psi P_\psi |\langle x,p|\psi\rangle|^2,$$

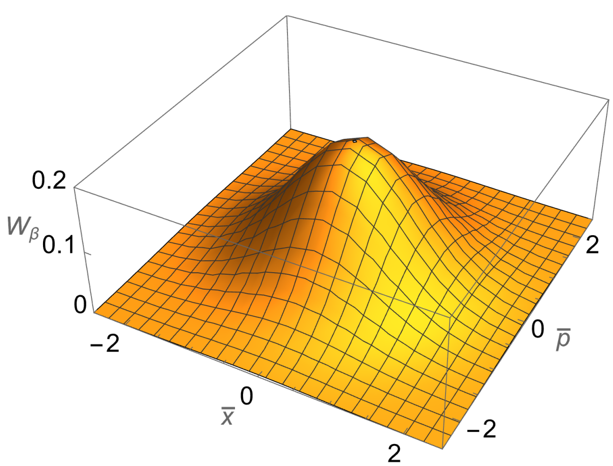
which is then also guaranteed to be normalized and non-negative.

Application to the photonic thermal state (at $\beta = 1/k_{\mathrm{B}}T$):

$$W_\beta(x,p) = \frac{e^{-|\gamma|^2}}{\pi}\sum_{n=0}^{\infty} P_n \frac{|\gamma|^{2n}}{n!}, \quad |\gamma|^2 = \bar{x}^2 + \bar{p}^2, \quad P_n = (1 - e^{-\beta\hbar\omega})\, e^{-\beta n\hbar\omega}.$$

Photonic thermal states are mixed quantum states composed of (stationary) number states. This makes the Husimi distribution time-independent.

The graph represents the function $W_\beta(x,p)$ for $\beta\hbar\omega = 1$.



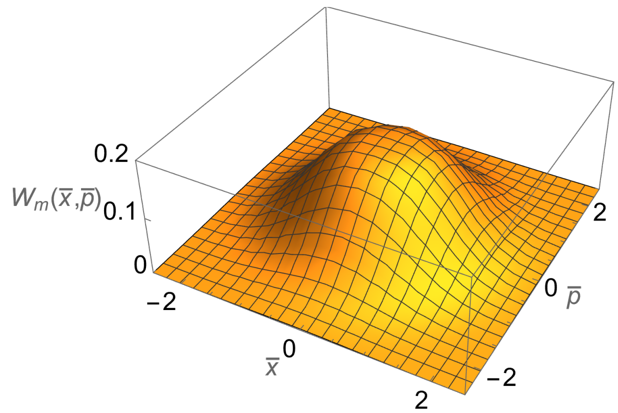
Consider the two photonic number states $|0\rangle$ and $|1\rangle$. Both are stationary states. Their Husimi distributions are time-independent.

If we combine them into a mixed state,

$$\rho_{\mathrm{m}} = \frac{1}{2}\big[|0\rangle\langle 0| + |1\rangle\langle 1|\big],$$

its Husimi distribution, $W_{\mathrm{m}}(x,p)$, is again time-independent [lex186].

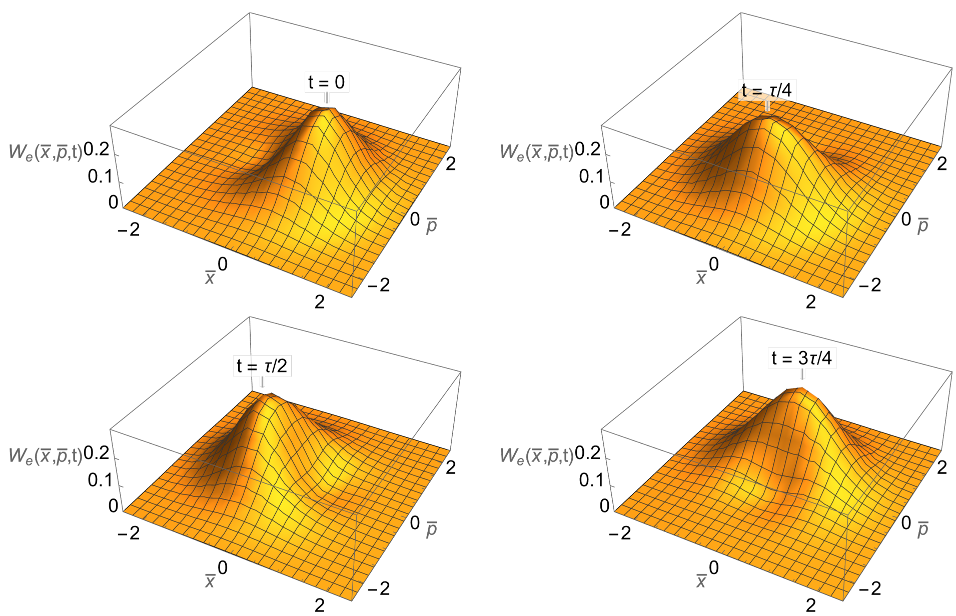
The graphs shows a central peak in the phase plane, which somewhat flattened and broadened relative to thar representing the state $|0\rangle$ alone.



If, on the other hand, we combine them into a pure quantum state,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big[|0\rangle + |1\rangle\big], \quad \rho_{\rm e} = |\psi\rangle\langle\psi|,$$

its Husimi distribution, $W_{\rm e}(x,p,t)$, is time-dependent [lex186].

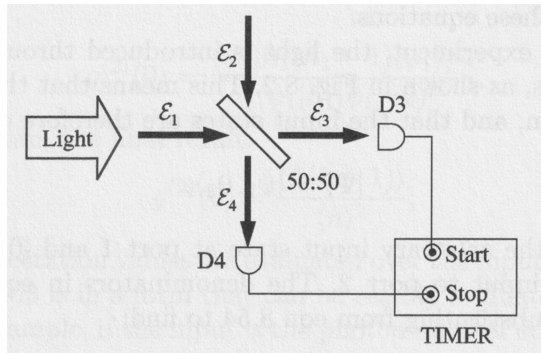


The four graphs are snapshots taken at successive time interval $\frac{1}{4}\tau$, where $\tau = 2\pi/\omega$. Taking the time average of $W_{\rm e}(x,p,t)$ produces $W_{\rm m}(x,p)$.

Intensity correlations of photonic number states:

Intensity interferometer of HBT experiment:[1]

- input ports 1,2
- output ports 3,4 with photon detectors,
- timer starts with D3 detection and stops with D4 detection,
- 50:50 beam splitter.



[image from Fox 2014]

Second-order correlation function for photonic number states:

$$g^{(2)}(\tau) = \frac{\langle n_3(t) n_4(t+\tau)\rangle}{\langle n_3(t)\rangle\langle n_4(t+\tau)\rangle} \xrightarrow{\tau\to 0} \frac{\langle a_3^\dagger a_4^\dagger a_4 a_3\rangle}{\langle a_3^\dagger a_3\rangle\langle a_4^\dagger a_4\rangle}.$$

The last step also involves normal ordering of operators.

Input electric fields: E_1, E_2.

Output electric fields:[2] $E_3 = \frac{1}{\sqrt{2}}(E_1 - E_2), \quad E_4 = \frac{1}{\sqrt{2}}(E_1 + E_2)$.

Harmonic oscillator representation of electric fields: $E \propto a^\dagger + a$.

Consequence of linearity: $a_3 = \frac{1}{\sqrt{2}}(a_1 - a_2), \quad a_4 = \frac{1}{\sqrt{2}}(a_1 + a_2)$, etc.

Input port 2 is shut down: $|\Psi\rangle = |n, 0\rangle$ (stream of photons from port 1).

Expectation values for photonic number states:

$$\langle a_3^\dagger a_3\rangle = \langle a_4^\dagger a_4\rangle = \frac{1}{2}\langle n\rangle, \quad \langle a_3^\dagger a_4^\dagger a_4 a_3\rangle = \frac{1}{4}\big[\langle n\rangle^2 - \langle n\rangle\big].$$

Evidence of antibunching: $g^{(2)}(0) = \frac{\langle n\rangle[\langle n\rangle - 1]}{\langle n\rangle^2} < 1$.

[1] R. Hanbury Brown and R. Q. Twiss are astronomers.

[2] The minus sign arises from a phase change in the 50:50 beam splitter.