Principle of Relativity [lam27]

The laws of physics must hold for all inertial reference frames. Hence predictions about measurable quantities made from different inertial reference frames can only depend on relative motion.

This weaker version of the principle of relativity dates back to the time of Galileo. It is our focus here. There is a stronger version enunciated by Einstein, which includes noninertial frames and inspired *general relativity*.

Newton's laws of mechanics are consistent with the (weaker) principle of relativity if we assume that distances and time intervals are the same in all inertial frames. The laws of electromagnetism are not.

If we wish to uphold the principle of relativity for electromagnetism, the structure of space and time has to be modified. This goal compelled Einstein
[lln16][lln25] to formulate the theory of *special relativity*, to be discussed later.

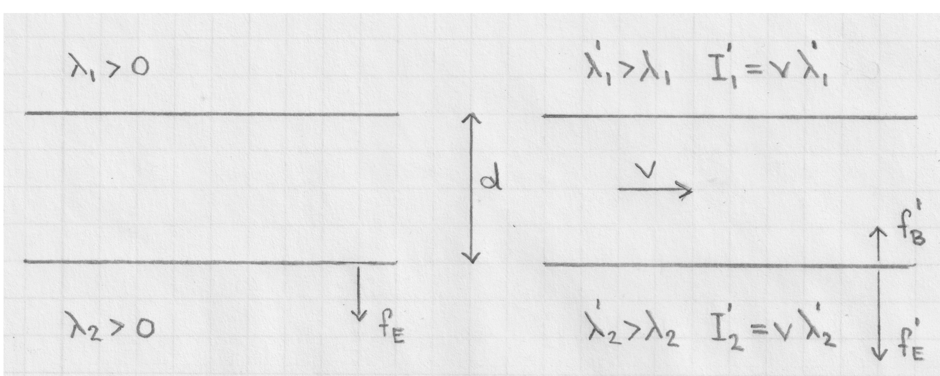
Here we take a look at two situations where the principle of relativity as applied to electromagnetism calls for a modified conception of space and time. Results established earlier in this course bring the issue to a head.

Electrically charged rails:

Consider two long, parallel, uniformly charged rails as shown.

One observer sees the two rails at rest (left) and measures a repulsive electric force between them. Another observer sees the rails in motion (right) and measures also an attractive magnetic force.

The principle of relativity demands that the net force measured by both observers must be the same.



- ▷ Electric field of rail 1 at position of rail 2 and (repulsive) electric force (per unit length) on rail 2 when both rails are at rest:

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda_1}{d}, \quad f_E = \lambda_2 E = \frac{1}{2\pi\epsilon_0}\frac{\lambda_1\lambda_2}{d}.$$

- ▷ Currents of moving rails: $I_1' = v\lambda_1', \quad I_2' = v\lambda_2'$.
- ▷ Magnetic field of rail 1 at position of rail 2 and (attractive) magnetic force (per unit length) on rail 2 when both rails are moving:

$$B' = \frac{\mu_0}{2\pi}\frac{I_1'}{d}, \quad f_B' = I_2'B' = \frac{\mu_0}{2\pi}\frac{I_1'I_2'}{d} = \frac{\mu_0 v^2}{2\pi}\frac{\lambda_1'\lambda_2'}{d}.$$

- ▷ Net force (per unit length) between moving rails:

$$f_E' - f_B' = \frac{1}{2\pi\epsilon_0}\frac{\lambda_1'\lambda_2'}{d} - \frac{\mu_0 v^2}{2\pi}\frac{\lambda_1'\lambda_2'}{d} = \frac{1}{2\pi\epsilon_0}\frac{\lambda_1'\lambda_2'}{d}\left(1 - \frac{v^2}{c^2}\right).$$

  In the last step we have used $\mu_0 = \frac{1}{\epsilon_0 c^2}$, where c is the speed of light.
- ▷ The principle of relativity is not satisfied if $f_E' - f_B'$ differs from f_E, which it does if we insist that distances and time intervals are absolute.
- ▷ The theory of relativity predicts that distances between any two points on a rail are contracted by a fraction $\sqrt{1 - v^2/c^2}$ when measured by the observer in relative motion to them.
- ▷ In consequence, the charge density on both rails is higher by the same fraction when observed in motion:

$$\lambda_1' = \frac{\lambda_1}{\sqrt{1 - v^2/c^2}}, \quad \lambda_2' = \frac{\lambda_2}{\sqrt{1 - v^2/c^2}} \quad \Rightarrow \lambda_1'\lambda_2' = \lambda_1\lambda_2\left(1 - \frac{v^2}{c^2}\right)^{-1}.$$

- ▷ Any relative motion between rails and observer not only produces a magnetic force, but also modifes the electric force. The two effects do not change the net force, thus uphold the principle of relativity:

$$f_E' - f_B' = \frac{1}{2\pi\epsilon_0}\frac{\lambda_1'\lambda_2'}{d}\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{2\pi\epsilon_0}\frac{\lambda_1\lambda_2}{d} = f_E.$$

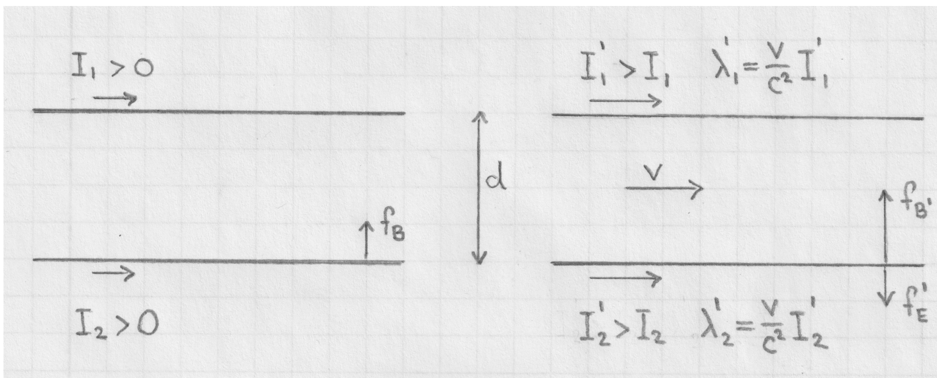
Current-carrying rails:

Consider two long, parallel rails carrying electric currents as shown.

One observer sees the two rails at rest and electrically neutral (left), measuring an attractive magnetic force between them.

Another observer sees the rails in motion and electrically charged (right), measuring a modified magnetic force and an electric force.

The principle of relativity again demands that the net force measured by both observers must be the same.



▷ Magnetic field of rail 1 at position of rail 2 and (attractive) magnetic force (per unit length) on rail 2 when both rails are at rest:

$$B = \frac{\mu_0}{2\pi}\frac{I_1}{d}, \quad f_B = I_2 B = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}.$$

[lln25] ▷ Currents in moving rails are relativistically enhanced:

$$I'_1 = \frac{I_1}{\sqrt{1 - v^2/c^2}}, \quad I'_2 = \frac{I_2}{\sqrt{1 - v^2/c^2}}.$$

▷ The enhancement is the same when the rails move the other way. The effect depends on the relative velocity between positive and negative charges inside the rails.[1]

[1]In the preceding module [lln11], we identified the electric current as a flux quantity associated with current density – here the net charge flowing past a cross sectional area of each rail per unit time. It's that time which is different for the two observers, owing to the relativistic effect of time dilation.

- ▷ The observer who sees the rails in motion thus predicts a stronger magnetic force between them:

$$f'_B = \frac{\mu_0}{2\pi}\frac{I'_1 I'_2}{d} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\frac{1}{1-v^2/c^2}.$$

- ▷ Current-carrying rails which are electrically neutral when seen at rest become electrically charged when seen in motion. The charge densities
[lln25] predicted by the theory of relativity are

$$\lambda'_1 = \frac{v}{c^2}\,I'_1 = \frac{(v/c^2)I_1}{\sqrt{1-v^2/c^2}}, \qquad \lambda'_2 = \frac{v}{c^2}\,I'_2 = \frac{(v/c^2)I_2}{\sqrt{1-v^2/c^2}}.$$

  The length contraction associated with the moving rails affects the densities of positive and negative charges differently when an electric current is present.[2]

- ▷ Both moving rails carry like charges and thus experience a repulsive electric force,

$$f'_E = \frac{1}{2\pi\epsilon_0}\frac{\lambda'_1\lambda'_2}{d} = \frac{1}{2\pi\epsilon_0 c^2}\frac{I_1 I_2}{d}\frac{(v/c)^2}{\left[\sqrt{1-v^2/c^2}\right]^2} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\frac{v^2/c^2}{1-v^2/c^2}.$$

- ▷ The net force is again the same for both observers, in agreement with the principle of relativity:

$$f'_B - f'_E = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\frac{1-v^2/c^2}{1-v^2/c^2} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = f_B.$$

A more systematic account of relativistic kinematics and dynamics will come
[lln16][lln25] along in later modules.

[2] A positive current I flowing to the right is associated with positive charge carriers drifting to the right and/or negative charge carriers drifting to the left. An observer who sees the rail moving to the right sees the density of mobile charge carriers increased if they are positive and decreased if they are negative, which makes the net charge density positive in both versions of this situation.