Doppler Effect [lam22]

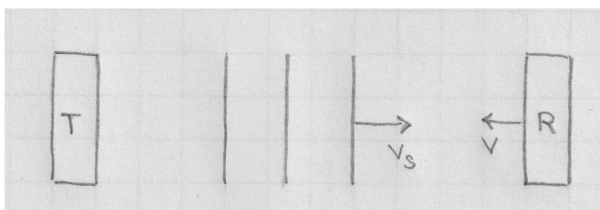
Doppler effect for sound:

Sound waves propagate with velocity v_s through medium (e.g. air).

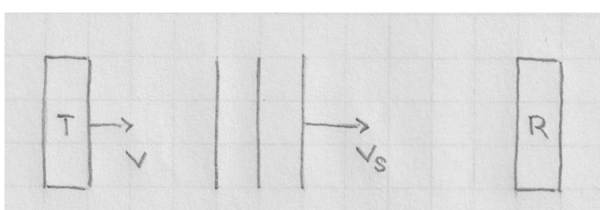
Transmitter T and receiver R are in relative motion with velocity $v < v_s$ toward each other. Their motion relative to the medium matters.

Time interval: Δt.
Transmitter frequency: ν_T.
Number of cycles transmitted in Δt: $N_T = \nu_T \Delta t$.
Number of cycles received in Δt: $N_R = \nu_R \Delta t$.

(a) Case when transmitter is at rest in medium:
Distance occupied by N_T cycles: $N_T \lambda = v_s \Delta t$.
Distance occupied by N_R cycles: $N_R \lambda = (v_s + v)\Delta t$.
Wavelength in medium: $\lambda = \dfrac{v_s}{\nu_T} = \dfrac{v_s + v}{\nu_R}$.
Frequency at receiver: $\nu_R = \nu_T \dfrac{v_s + v}{v_s} = \nu_T(1 + v/v_s)$.



(b) Case when receiver is at rest in medium:
Distance occupied by N_T cycles: $N_T \lambda = (v_s - v)\Delta t$.
Distance occupied by N_R cycles: $N_R \lambda = v_s \Delta t$.
Wavelength in medium: $\lambda = \dfrac{v_s - v}{\nu_T} = \dfrac{v_s}{\nu_R}$.
Frequency at receiver: $\nu_R = \nu_T \dfrac{v_s}{v_s - v} = \dfrac{\nu_T}{1 - v/v_s} \overset{v \ll v_s}{\rightsquigarrow} \nu_T(1 + v/v_s)$.



For velocities that are small compared to the speed of sound, $v \ll v_s$, only the relative motion between transmitter and receiver matters.

Doppler effect for light:

Transmitter T and receiver R are in relative motion with velocity v toward each other.

Electromagnetic waves travel at speed c relative to the transmitter T and relative to the receiver R, irrespective of their relative motion.

There is no medium for electromagnetic waves to which a state of motion could be attributed.

Proper time interval measured in T-frame: Δt_T.

Dilated time interval measured in R-frame: $\Delta t_R = \dfrac{\Delta t_T}{\sqrt{1 - v^2/c^2}}$.

Frequency of transmitter: ν_T.

Number of cycles transmitted in Δt_T: $N_T = \nu_T \Delta t_T$.

Distance occupied by N_T cycles in T-frame: $N_T \lambda_T = c\Delta t_T$.

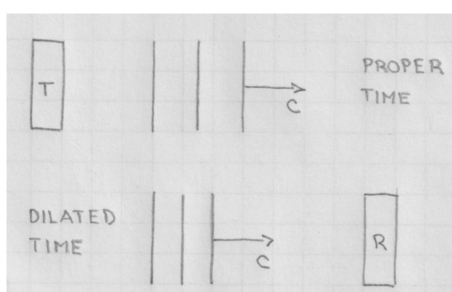
Wavelength in T-frame: $\lambda_T = \dfrac{c\Delta t_T}{N_T} = \dfrac{c}{\nu_T}$.

Distance occupied by N_T cycles in R-frame: $N_T \lambda_R = (c - v)\Delta t_R$.

Wavelength in R-frame:

$$\lambda_R = \frac{c - v}{N_T}\,\Delta t_R = \frac{c - v}{\nu_T}\frac{\Delta t_R}{\Delta t_T} = \frac{c}{\nu_T}\frac{1 - v/c}{\sqrt{1 - v^2/c^2}} = \frac{c}{\nu_T}\sqrt{\frac{1 - v/c}{1 + v/c}}.$$

Frequency in R-frame: $\nu_R = \dfrac{c}{\lambda_R} = \nu_T\sqrt{\dfrac{1 + v/c}{1 - v/c}} \overset{v\ll c}{\rightsquigarrow} \nu_T(1 + v/c)$..



For velocities that are small compared to the speed of light, $v \ll c$, the expression of the Doppler effect for sound is recovered. [lex144]