Mesoscopic Fields and Sources [lam20]

Classical electrodynamics – beginning with electrostatics and magnetostatics – employs scalar and vector fields for the description of most everything.

This choice is in tension with the facts that (i) matter is composed of particles and (ii) the electric and magnetic properties of matter are derived from those of particles. The particles in question are elementary (electrons, protons, neutrons) or composite (atoms, ions, molecules).

[lln9] This tension first comes to a head in the dielectric response of matter to an electrostatic field.

It is useful to introduce three distinct length scales $d \ll L \ll R$:

- The *microscopic* length scale d roughly measures the size of or average distance between the relevant particles.
- The *mesoscopic* length scale L is the span of averaging done to produce continuous scalar and vector fields from the effects of particles.
- The *macroscopic* length scale R is the size of the dielectric (or conducting or magnetic) material under scrutiny.

Relevant fields in the electrostatics of dielectric materials are the scalars ρ (charge density), Φ (electric potential) and the vectors $\mathbf{E}$ (electric field), $\mathbf{D}$ (displacement field), $\mathbf{P}$ (polarization).

On a microscopic length scale, they are all wildly varying in magnitude and (if applicable) direction. The quantities employed in [lln9] are averages inferred from convolution integrals in the form

$$\psi(\mathbf{x})_{\mathrm{mes}} = \int d^3x'\, \psi(\mathbf{x}')_{\mathrm{mic}} f_L(\mathbf{x}-\mathbf{x}'), \qquad \int d^3x\, f_L(\mathbf{x}) = 1,$$

where the function $f_L(\mathbf{x})$ has a plateau structure over a distance $\sim L$ and smooth edges over a similar distance. The switch from $\psi(\mathbf{x})_{\mathrm{mic}}$ to $\psi(\mathbf{x})_{\mathrm{mes}}$ is tacitly understood in [lln9] without change in notation.

It is important that this kind of averaging commutes with spatial derivatives:

$$\int d^3x'\, \frac{\psi(\mathbf{x}')_{\mathrm{mic}}}{\partial x_i'} f_L(\mathbf{x}-\mathbf{x}') = -\int d^3x'\, \psi(\mathbf{x}')_{\mathrm{mic}} \frac{\partial f_L(\mathbf{x}-\mathbf{x}')}{\partial x_i'}$$
$$= \int d^3x'\, \psi(\mathbf{x}')_{\mathrm{mic}} \frac{\partial f_L(\mathbf{x}-\mathbf{x}')}{\partial x_i} = \frac{\partial}{\partial x_i}\, \psi(\mathbf{x})_{\mathrm{mes}}.$$