PHY455
Spring, 2020
Exam #2

**Name** _____

**Total** _____

All answers should be given in eV or Å and related units where appropriate, and SI unless otherwise specified. All answers should be given numerically wherever possible unless otherwise stated.

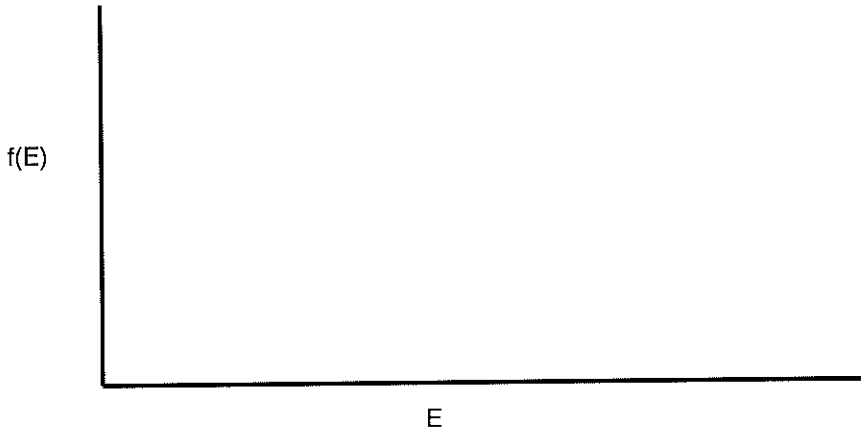
Show your work.

Exam2S20

1. Derive an expression for the density of states for the 2D free electron gas. (**10**)

2. Find the KE for the 2D free electron gas at T = 0. Give your answer in terms of the area of the crystal and $\varepsilon_F$ (and the usual constants). (**10**)

3. a. Sketch a. D(ε) and b. f(ε) vs. ε for a 3D electron gas at T~3000K. Indicate the approximate location of the Fermi Energy in both cases. (**10**)

4. For the electron gas, find the occupation probability for a state 0.300 eV a. below and b. above the chemical potential for a temperature of 3000K. (**10**)

5. a. Calculate the heat capacity per T at low T of 1 mole of potassium atoms due to free electrons (3D) and compare to the experimental value. (**15**)

Hint: $C_v = \frac{\pi^2}{3} k_B^2 T D(\varepsilon_F)$ ; $D(\varepsilon) = \frac{3}{2}\frac{N}{\varepsilon}$

6. Assume atoms of valency 2 are arranged in a BCC lattice with (cubic) lattice constant 3.50 Å. (Hint: this is not the nearest-neighbor spacing.) Calculate:
a. atomic number density (SI units) (**5**)

b. valence electron concentration (SI units) (**5**)

c. Fermi wavenumber (SI units) (**5**)

d. Fermi energy (eV) (**5**)

f. Fermi temperature (**5**)

g. $r_s$ parameter (Å) (**5**)

h. Fermi velocity (SI units) (**5**)

i. Relaxation time assuming a mean free path of 1.00 μm. (**5**)

j. mobility (SI units) (**5**)

k. electrical conductivity (SI units) (**5**)

7. For a simple cubic lattice with cube edge a = 4.00 Å, the valency is 3.
a. Find the radius of the Fermi sphere in k space. (SI units) (**5**)

b. Compare the volume of the Fermi sphere to the volume of the 1st Brillouin zone. (**5**)

c. Will the Fermi sphere fit into the 1st BZ? Explain. (**5**)

8. For the simple cubic crystal of side a, give the **G** vector necessary to reduce the following **k** vectors into the 1st Brillouin Zone. Give the resultant **k** vector. Give the **k** and **G** vectors in terms of a and $\prod$. (**15**)

a. $\pi$/a[010]

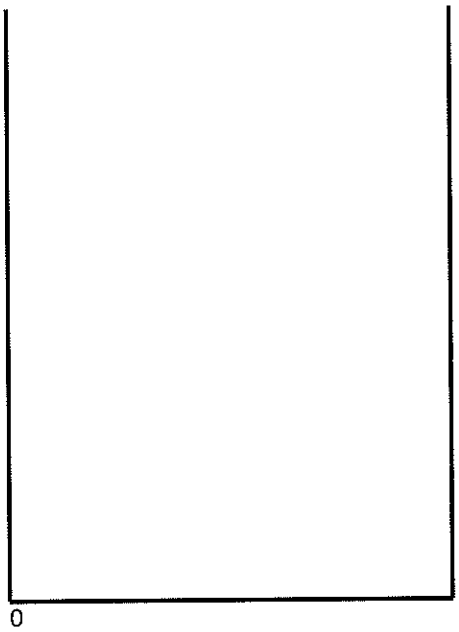
b. $7\pi$/a[110]

c. $9\pi$/4a[111]

9. For a 2D square lattice with lattice constant a, fill in the table and plot 6 of the slowest lying sets of bands (in energy) in the [10] direction. Find the energy of the lowest band at the zone boundary (in terms of ∏, m, a, and ħ). and call that $E_o$.

Label the various energies in terms of $E_o \equiv \frac{\hbar^2}{2m}(\frac{\pi}{a})^2$ and the k values in terms of ∏ and a. Generate the table below, then sketch the bands. Label the axes and bands and plot the bands below. (**25**)

| Band #'s | G vector | E(0) | E(zone boundary) |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |



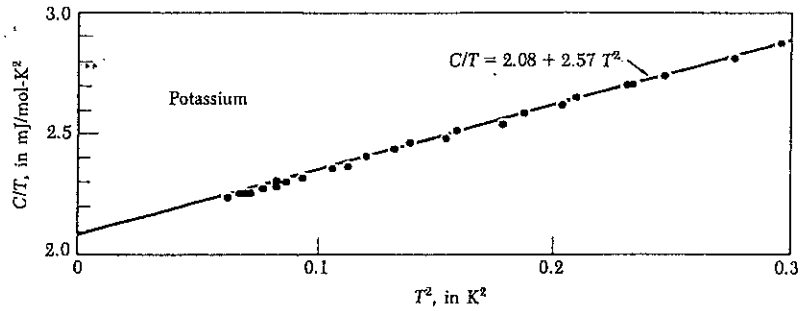



Figure 9 Experimental heat capacity values for potassium, plotted as $C/T$ versus $T^2$. (After W. H. Lien and N. E. Phillips.)

Table 1 Calculated free electron Fermi surface parameters for metals at room temperature

(Except for Na, K, Rb, Cs at 5 K and Li at 78 K)

| Valency | Metal | Electron concentration, in $cm^{-3}$ | Radius[a] parameter $r_s$ | Fermi wavevector, in $cm^{-1}$ | Fermi velocity, in cm $s^{-1}$ | Fermi energy, in eV | Fermi temperature $T_F = \epsilon_F/k_B$, in deg K |
|---|---|---|---|---|---|---|---|
| 1 | Li | $4.70 \times 10^{22}$ | 3.25 | $1.11 \times 10^8$ | $1.29 \times 10^8$ | 4.72 | $5.48 \times 10^4$ |
| | Na | 2.65 | 3.93 | 0.92 | 1.07 | 3.23 | 3.75 |
| | K | 1.40 | 4.86 | 0.75 | 0.86 | 2.12 | 2.46 |
| | Rb | 1.15 | 5.20 | 0.70 | 0.81 | 1.85 | 2.15 |
| | Cs | 0.91 | 5.63 | 0.64 | 0.75 | 1.58 | 1.83 |
| | Cu | 8.45 | 2.67 | 1.36 | 1.57 | 7.00 | 8.12 |
| | Ag | 5.85 | 3.02 | 1.20 | 1.39 | 5.48 | 6.36 |
| | Au | 5.90 | 3.01 | 1.20 | 1.39 | 5.51 | 6.39 |
| 2 | Be | 24.2 | 1.88 | 1.93 | 2.23 | 14.14 | 16.41 |
| | Mg | 8.60 | 2.65 | 1.37 | 1.58 | 7.13 | 8.27 |
| | Ca | 4.60 | 3.27 | 1.11 | 1.28 | 4.68 | 5.43 |
| | Sr | 3.56 | 3.56 | 1.02 | 1.18 | 3.95 | 4.58 |
| | Ba | 3.20 | 3.69 | 0.98 | 1.13 | 3.65 | 4.24 |
| | Zn | 13.10 | 2.31 | 1.57 | 1.82 | 9.39 | 10.90 |
| | Cd | 9.28 | 2.59 | 1.40 | 1.62 | 7.46 | 8.66 |
| 3 | Al | 18.06 | 2.07 | 1.75 | 2.02 | 11.63 | 13.49 |
| | Ga | 15.30 | 2.19 | 1.65 | 1.91 | 10.35 | 12.01 |
| | In | 11.49 | 2.41 | 1.50 | 1.74 | 8.60 | 9.98 |
| 4 | Pb | 13.20 | 2.30 | 1.57 | 1.82 | 9.37 | 10.87 |
| | Sn($\omega$) | 14.48 | 2.23 | 1.62 | 1.88 | 10.03 | 11.64 |

[a]The dimensionless radius parameter is defined as $r_s = r_0/a_H$, where $a_H$ is the first Bohr radius and $r_0$ is the radius of a sphere that contains one electron.