

Your Name: \_\_\_\_\_

PHY203

Final Exam  
5/11/09

## Part 3

# Solutions

1. Masses are placed as follows along the x-axis: 1kg at x=0, 2kg at x=1m, and 3kg at x=2m, all connected by a massless rod. Find the x position of the center of mass:

- a. **1.33m**                       $x_{cm} = [(1\text{kg})(0\text{m}) + (2\text{kg})(1\text{m}) + (3\text{kg})(2.0\text{m})]/(6\text{kg})$   
 b. 1.6m                               $= [8\text{kgm}]/(6\text{kg})$   
 c. 8.0m  
 d. 14.0m  
 e. none of the above

2. Masses are placed as follows along the x-axis: 1kg at x=0, 2kg at x=1m, and 3kg at x=2m, all connected by a 5kg rod of length 3m starting at x=0. Find the x position of the center of mass:

- a. **1.4m**                               $x_{cm} = [(1\text{kg})(0\text{m}) + (2\text{kg})(1\text{m}) + (3\text{kg})(2.0\text{m}) +$   
 b. 1.55m                               $(5\text{kg})(1.5\text{m})]/(11\text{kg})$   
 c. 2.1m                                 $= [15.5\text{kgm}]/(11\text{kg})$   
 d. 15.5m  
 e. 23.0m

For problems 3 and 4:

A 0.25kg ball traveling at a speed of 3.5m/s in the positive x-direction strikes a wall and rebounds in the negative x-direction with a speed of 2.5m/s.

3. Find the magnitude of the impulse.

- a. 0.25Ns                               $I = \Delta p = |(0.25\text{kg})(-2.5\text{m/s} - 3.5\text{m/s})|$   
 b. 1.0Ns                                 $= (0.25\text{kg})(6.0\text{m/s})$   
 c. **1.5Ns**  
 d. 6.0Ns  
 e. 24.0Ns

4. Assuming the collision time is  $1.0 \times 10^{-3}\text{s}$ , find the magnitude of the average force during the collision.

- a.  $2.5 \times 10^{-4}\text{N}$                        $I = \Delta p = F_{av} \Delta t$   
 b.  $1.5 \times 10^{-3}\text{N}$                        $1.5\text{Ns} = F_{av}(1.0 \times 10^{-3}\text{s})$   
 c.  $2.5 \times 10^{+2}\text{N}$   
 d.  $1.0 \times 10^{+3}\text{N}$   
 e.  **$1.5 \times 10^{+3}\text{N}$**

5. Let the vector  $\mathbf{A} = 5\mathbf{i} - 4\mathbf{j}$  and  $\mathbf{B} = -3\mathbf{i} + 2\mathbf{k}$ . Find the vector  $\mathbf{A} \times \mathbf{B}$ :

- a.  **$-8\mathbf{i} - 10\mathbf{j} - 12\mathbf{k}$**                        $\mathbf{A} \times \mathbf{B} = (5\mathbf{i} - 4\mathbf{j}) \times (-3\mathbf{i} + 2\mathbf{k}) = -15(\mathbf{i} \times \mathbf{i}) + 10(\mathbf{i} \times \mathbf{k})$   
 b.  $-8\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$                        $+ 12(\mathbf{j} \times \mathbf{i}) - 8(\mathbf{j} \times \mathbf{k})$   
 c.  $-8\mathbf{i} + 10\mathbf{j} - 12\mathbf{k}$                        $= 0 - 10\mathbf{j} - 12\mathbf{k} - 8\mathbf{i}$   
 d.  $-8\mathbf{i} + 10\mathbf{j} + 12\mathbf{k}$

e.  $+8i + 10j + 12k$

For problems 6-8:

A solid sphere of mass 10kg and radius 0.5m is at rest. A force of 10N is applied at the edge of the sphere in a direction perpendicular to the radius.

6. Find the magnitude of the torque,  $t$  about the central axis of the sphere:

a. 0 Nm  $t = rF\sin\theta = (0.5m)(10N)(\sin 90^\circ)$

**b. 5.0 Nm**

c. 10.0 Nm

d. 50.0 Nm

e. 100.0 Nm

7. Find the moment of inertia about the axis of the sphere:

a.  $0.25 \text{ kgm}^2$   $I = \frac{2}{5}Mr^2 = \frac{2}{5}(10\text{kg})(0.5\text{m})^2$

**b.  $1.0 \text{ kgm}^2$**

c.  $1.67 \text{ kgm}^2$

d.  $2.0 \text{ kgm}^2$

e.  $4.0 \text{ kgm}^2$

8. Find the number of revolutions the sphere has made after the torque has been applied for 15s.

a. 11.9  $q = q_0 + \omega_0 t + \frac{1}{2}at^2$

b. 75.0  $= 0 + 0 + \frac{1}{2}(t/I)(15\text{s})^2$

**c. 89.5**  $= \frac{1}{2}(5\text{Nm}/1 \text{ kgm}^2)(15\text{s})^2$

d. 179.0  $= 562.5\text{rad}$

e. 562.5  $\#rev = 562.5/(2\pi)$

For problems 9 and 10:

A hoop (thin cylindrical shell) of mass 2kg and radius 0.5m is rotating horizontally on a frictionless table with an angular speed of 5rad/s.

9. Find the magnitude of the angular momentum:

a.  $1.25\text{kgm}^2/\text{s}$   $L = I\omega = (mr^2)\omega = (2\text{kg})(0.5\text{m})^2 (5\text{rad/s})$

**b.  $2.5\text{kgm}^2/\text{s}$**

c.  $5.0\text{kgm}^2/\text{s}$

d.  $15.7\text{kgm}^2/\text{s}$

e. none of the above

10. A blob of clay of mass 1.0kg is dropped onto the rim of the hoop from above and sticks to the hoop. Find the magnitude of the angular speed of the hoop-putty system after the putty has stuck to the hoop:

a.  $0.833\text{rad/s}$   $2.5\text{kgm}^2/\text{s} = I_{\text{hoop}} + mr^2 = (0.5\text{kgm}^2 + (1\text{kg})(0.5\text{m})^2)\omega$

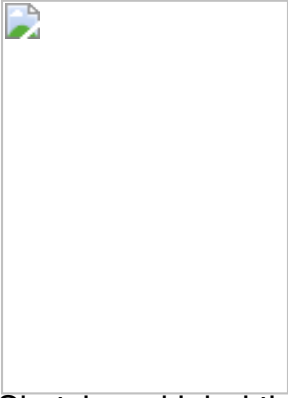
b.  $2.5\text{rad/s}$   $= (0.75\text{kgm}^2)\omega$

**c.  $3.33\text{rad/s}$**

d.  $5.0\text{rad/s}$

e.  $6.67\text{rad/s}$

11. Two masses are supported by a pulley, which can be modeled as a solid cylinder, as shown below. The values are as follows:  $m_1=4\text{kg}$ ,  $m_2=5\text{kg}$ ,  $M_{\text{pulley}}=7\text{kg}$ ,  $R_{\text{pulley}}=0.5\text{m}$ . (Use  $g=9.81 \text{ m/s}^2$ .)



- a. Sketch and label the forces on the two masses on the picture above. (Hint: the tensions in the rope on either side are not equal in magnitude.)
- b. Using the coordinate systems shown above, write down Newton's 2nd Law for the two masses in terms of  $g$ , the masses, the tensions, and the acceleration,  $a$ .

$$1: T_1 - m_1g = m_1a$$

$$2: -T_2 + m_2g = m_2a$$

- c. Write down the torque equation for the pulley in terms of  $M_{\text{pulley}}, R_{\text{pulley}}$ , the tensions, and the acceleration,  $a$ .

$$3: \tau = I\alpha = (1/2MR^2)(a/R) = RT_2 - RT_1$$

- d. Combine the results of parts b-d to find the acceleration of the masses.

$$\text{Cancel the R's in 3 eq. 3: } T_2 - T_1 = 1/2Ma$$

Add this with eqs. 1 and 2:

$$m_2g - m_1g = m_1a + m_2a + 1/2Ma$$

$$a = (m_2 - m_1)g / (m_1 + m_2 + 1/2M)a$$

$$= (5\text{kg} - 4\text{kg})(9.81\text{m/s}^2) / (4\text{kg} + 5\text{kg} + 7/2\text{kg}) = 0.78\text{m/s}^2$$