

Your Name: _____

PHY203

Exam #3
Chapters 8-10
Fri., 4/16/10

Solutions

For problems 1-3,

Point masses are placed as follows along the x-axis: 1kg at x=0, 2kg at x=1m, and 3kg at x=2m, all connected by a massless rod.

1. Find the x position of the center of mass:

a. 0 $x_{cm} = [(2\text{kg})(1\text{m}) + (3\text{kg})(2.0\text{m})]/(6\text{kg})$

b. 1.33m

c. 1.5m

d. 1.6m

e. 2.33m

2. Find the moment of inertia about the x-axis:

a. **0 kgm²** all mass on x-axis

b. 8.0 kgm²

c. 9.0 kgm²

d. 14.0 kgm²

e. 15.0 kgm²

3. Find the moment of inertia about the y-axis:

a. 0 kgm² $I = (1\text{kg})(0) + (2\text{kg})(1\text{m})^2 + (3\text{kg})(2\text{m})^2$

b. 8.0 kgm²

c. 9.0 kgm²

d. 14.0 kgm²

e. 15.0 kgm²

For problems 4 and 5,

4. A hoop of mass 4kg and radius 1.0m is rotating with an angular speed of 3rad/s. A blob of clay of mass 0.5kg is dropped onto the rim of the hoop from above and sticks to the hoop. Find the magnitude of the angular momentum of the hoop-putty system before the putty has stuck to the hoop:

a. 33.0 kgm²/s $L = lw = (4.0\text{kg})(1.0\text{m})^2(3\text{rad/s})$

b. 4.0 kgm²/s

c. 6.0 kgm²/s

d. 12.0 kgm²/s

e. 48.0 kgm²/s

5. Find the magnitude of the angular velocity of the hoop-putty system after the putty has stuck to the hoop:

- a. 0.89 rad/s
b. 2.67 rad/s
 c. 3.0 rad/s
 d. 3.43 rad/s
 e. 10.7 rad/s

$$L=I\omega; I_1\omega_1 = I_2\omega_2$$

$$12.0 \text{ kgm}^2/\text{s} = ((4.0\text{kg})(1.0\text{m})^2 + (0.5\text{kg})(1.0\text{m})^2 \text{ kgm}^2) (\omega_2)$$

$$=(4.5 \text{ kgm}^2/\text{s}) (\omega_2)$$

6. A hoop is lying on a horizontal table while it is spinning. Looking down on the table from above, the hoop is spinning in a counterclockwise direction. Find the direction of the angular momentum vector:
a. Out of the table (towards the observer) use right hand rule
 b. Into the table (away from the observer)
 c. Neither a. nor b.
 d. Not enough information is given.

7. Let the vector $\mathbf{A} = -4\mathbf{j}$ and $\mathbf{B} = 3\mathbf{j}$. Find the vector $\mathbf{A} \times \mathbf{B}$:

- a. 0** $\mathbf{A} \times \mathbf{B} = (-4\mathbf{j}) \times (3\mathbf{j}) = -12(\mathbf{j} \times \mathbf{j}) = 0$
 b. $-12\mathbf{j}$
 c. $+12\mathbf{i}$
 d. $-12\mathbf{k}$
 e. $+12\mathbf{k}$

8. Let the vector $\mathbf{A} = -4\mathbf{j}$ and $\mathbf{B} = 3\mathbf{i}$. Find the vector $\mathbf{A} \times \mathbf{B}$:

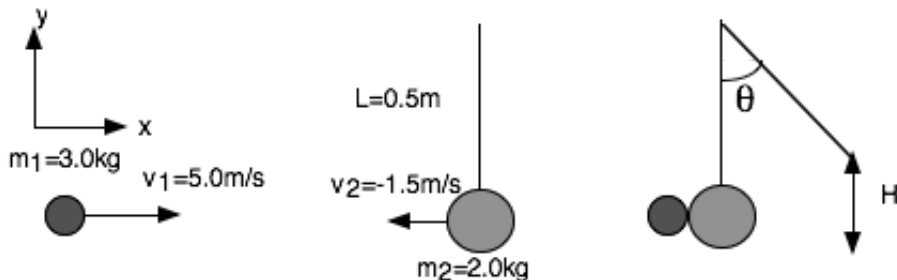
- a. 0** $\mathbf{A} \times \mathbf{B} = (-4\mathbf{j}) \times (3\mathbf{i}) = -12(\mathbf{j} \times \mathbf{i}) = 12\mathbf{k}$
 b. $-12\mathbf{i}$
 c. $+12\mathbf{i}$
 d. $-12\mathbf{k}$
 e. $+12\mathbf{k}$

9. Let the vector $\mathbf{A} = 5\mathbf{i} - 4\mathbf{j}$ and $\mathbf{B} = -3\mathbf{i} + 2\mathbf{k}$. Find the vector $\mathbf{A} \times \mathbf{B}$:

- a. $-8\mathbf{i} - 10\mathbf{j} - 12\mathbf{k}$** $\mathbf{A} \times \mathbf{B} = (5\mathbf{i} - 4\mathbf{j}) \times (-3\mathbf{i} + 2\mathbf{k})$
 b. $-8\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$ $= -15(\mathbf{i} \times \mathbf{i}) + 10(\mathbf{i} \times \mathbf{k}) + 12(\mathbf{j} \times \mathbf{i}) - 8(\mathbf{j} \times \mathbf{k}) -$
 c. $-8\mathbf{i} + 10\mathbf{j} - 12\mathbf{k}$ $= 15(0) + 10(-\mathbf{j}) + 12(-\mathbf{k}) - 8\mathbf{i}$
 d. $-8\mathbf{i} + 10\mathbf{j} + 12\mathbf{k}$
 e. $+8\mathbf{i} + 10\mathbf{j} + 12\mathbf{k}$

10. Find the kinetic energy of a solid cylinder with mass=3kg and radius=2m which is rolling without slipping with a center-of-mass velocity of 4 m/s:

- a. 6 J** $K = (1/2)mv^2 + (1/2)I\omega^2 = (1/2)mv^2 + (1/2)(1/2)mr^2(v/r)^2$
 b. 12 J $= (1/2 + 1/4)mv^2 = (3/4)(3\text{kg})(4\text{m/s})^2$
 c. 24 J
d. 36 J
 e. 48 J



11. A 2.0kg pendulum bob at the end of a string of length $L=0.5\text{m}$ is released from some angle such that just as it reaches the bottom of its trajectory it has a speed of 1.5m/s. At this point, the 2.0 kg bob collides with a ball of mass 3.0kg and speed 5.0m/s, as shown above. The two balls stick together and swing up to the right. Use $g=9.81\text{m/s}^2$.

a. Calculate the initial linear momentum of the two ball system just before the collision and express it in vector notation using the coordinate system above.

$$\mathbf{p}_i = (3\text{kg})(5\text{m/s}) + (2\text{kg})(-1.5\text{m/s}) = 12\text{kgm/s}$$

b. Calculate the initial kinetic energy of the two-ball system just before the collision.

$$K_i = (1/2)mv^2 = 1/2(3\text{kg})(5\text{m/s})^2 + 1/2(2\text{kg})(-1.5\text{m/s})^2 = 39.8\text{J}$$

c. Calculate the linear momentum of the two-ball system just after the collision before the balls start to swing up and express it in vector notation using the coordinate system above.

$$\text{same: } \mathbf{p}_f = 12\text{kgm/s}$$

d. Calculate the velocity of the two-ball system just after the collision before the balls start to swing up and express it in vector notation using the coordinate system above.

$$\mathbf{p}_f = 12\text{kgm/s} = (5\text{kg})\mathbf{v}$$

$$\mathbf{v} = 2.4 \text{ m/s}$$

e. Calculate the kinetic energy of the two-ball system just after the collision before the balls start to swing up.

$$K_i = (1/2)mv^2 = 1/2(5\text{kg})(2.4\text{m/s})^2 = 14.4\text{J}$$

f. Is the collision elastic or inelastic or can you not tell?

inelastic (KE is not conserved)

g. Calculate the height, H, of the two-ball system when it stops (momentarily) at the highest point.

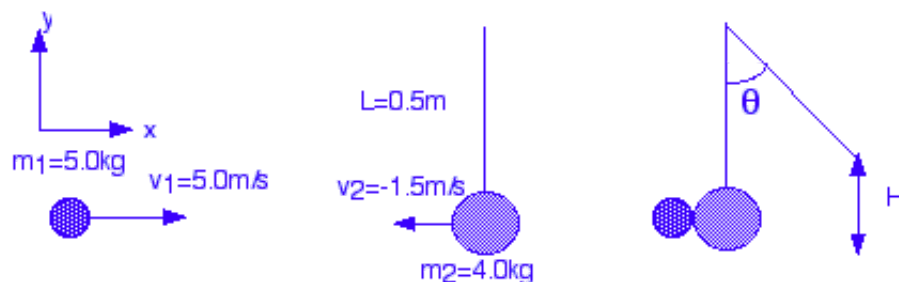
$$14.4\text{J} = mgh$$

$$h = 14.4/(5\text{kg})(9.81\text{m/s}^2) = 0.29\text{m}$$

h. Find the angle, θ , the string makes with the vertical at that height, H.

$$\cos\theta = (L-H)/L = (0.5\text{m} - 0.29\text{m})/(0.5\text{m}) = 0.42$$

$$\theta = \cos^{-1}(0.42) = 65^\circ$$



11. A 4.0kg pendulum bob at the end of a string of length $L=0.5\text{m}$ is released from some angle such that just as it reaches the bottom of its trajectory it has a speed of 1.5m/s . At this point, the 4.0kg bob collides with a ball of mass 5.0kg and speed 5.0m/s , as shown above. The two balls stick together and swing up to the right. Use $g=9.81\text{m/s}^2$.

a. Calculate the initial linear momentum of the two ball system just before the collision and express it in vector notation using the coordinate system above.

$$\mathbf{p}_i = (5\text{kg})(5\text{m/s}) + (4\text{kg})(-1.5\text{m/s}) = 19\text{kgm/s}$$

b. Calculate the initial kinetic energy of the two-ball system just before the collision.

$$K_i = (1/2)mv^2 = 1/2(5\text{kg})(5\text{m/s})^2 + 1/2(4\text{kg})(-1.5\text{m/s})^2 = 67.0\text{J}$$

c. Calculate the linear momentum of the two-ball system just after the collision before the balls start to swing up and express it in vector notation using the coordinate system above.

$$\text{same: } \mathbf{p}_f = 19\text{kgm/s}$$

d. Calculate the velocity of the two-ball system just after the collision before the balls start to swing up and express it in vector notation using the coordinate system above.

$$\mathbf{p_f} = 19\text{kgm/si} = (9\text{kg})\mathbf{v}$$

$$\mathbf{v} = 2.1 \text{ km/si}$$

e. Calculate the kinetic energy of the two-ball system just after the collision before the balls start to swing up.

$$K_i = (1/2)mv^2 = 1/2(9\text{kg})(2.1\text{m/s})^2 = 20.1\text{J}$$

f. Is the collision elastic or inelastic or can you not tell?

inelastic (KE is not conserved)

g. Calculate the height, H, of the two-ball system when it stops (momentarily) at the highest point.

$$20.1\text{J} = mgh$$

$$h = 20.1/(9\text{kg})(9.81\text{m/s}^2) = 0.23\text{m}$$

h. Find the angle, \mathbf{q} , the string makes with the vertical at that height, H.

$$\cos\mathbf{q} = (L-H)/L = (0.5\text{m} - 0.23\text{m})/(0.5\text{m}) = 0.54$$

$$\mathbf{q} = \cos^{-1}(0.54) = 57.3^\circ$$