Your Name: ____________________

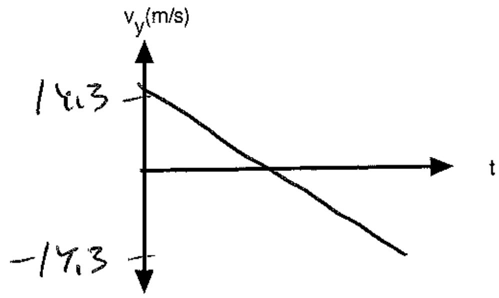
PHY203
Final Exam
5/8/19

- Show work
- Use correct SI units
- Use scientific notation
- All answers with 3 significant figures
- use $g = 9.81\ m/s^2$

Solutions

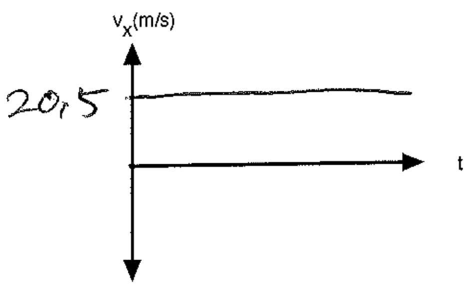
1. A ball is thrown from ground level with a speed of 25.0 m/s at an angle of 35.0° with respect to the horizontal. (Ignore air resistance.) Take the +y direction as the "up" direction. **Add numerical values to the vertical axes** (<u>for a.-d. only</u>).

a. Make a sketch of v_y vs. time while the ball is in the air.



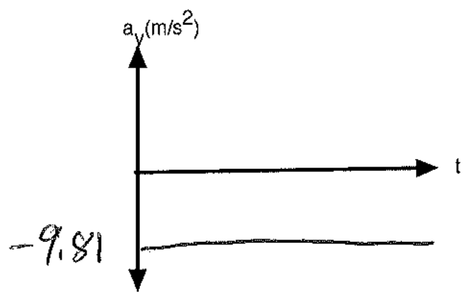
4

b. Make a sketch of v_x vs. time while the ball is in the air.



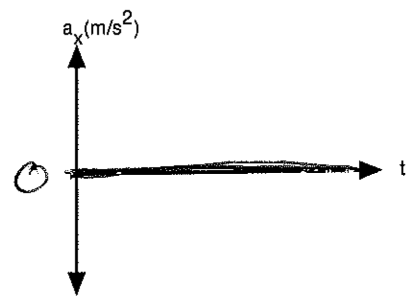
4

c. Make a sketch of a_y vs. time while the ball is in the air.



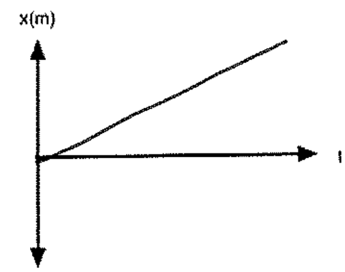
4

d. Make a sketch of a_x vs. time while the ball is in the air.



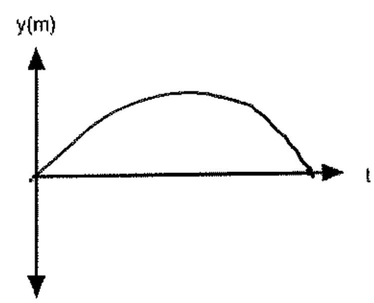
4

e. Make a sketch of x vs. time while the ball is in the air.



4

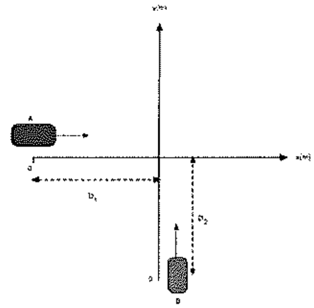
f. Make a sketch of y vs. time while the ball is in the air.



4

$v_{y0} = (25.0)\sin 35.0° = 14.3$ m/s

$v_{x0} = (25.0)\cos 35° = 20.5$ m/s



2. Two trains travel on perpendicular tracks. At a time of t=0 train A passes the x=0 point with a constant speed of 16.0 m/s. At a distance of D_1 = 215 m train A will cross the track that train B travels on. Train B starts at rest at y=0 at a distance of D_2 = 185 m from the point at which the tracks cross. Train B starts traveling in the +y direction with a constant acceleration of 4.50 m/s^2 at a time of t_B.

a. Using the coordinate system depicted above, write an equation of motion (x vs. t) for train A:

$x_A = 16.0t$ 5

b. Using the coordinate system depicted above, write an equation of motion (y vs. t) for train B:

$y_B = \frac{1}{2}(4.50)(t-t_B)^2 = 2.25(t-t_B)^2$ 11

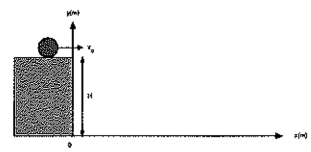
c. Find the time, t_B, such that the centers of the trains collide.

$16.0t = 215 \quad , \quad t = 13.4$ 15

$2.25(13.4-t_B)^2 = 185$

$(13.4-t_B)^2 = 82.2$

$t_B = 4.33\ s$



3. A ball with an initial speed of v_o=25.5 m/s slides off a horizontal, frictionless cliff at a height of H. It takes 17.5 s for the ball to hit the ground

a. Find the velocity and acceleration of the ball just after it slides off the cliff in vector notation.

$\vec{v} = 25.5 \frac{m}{s} \hat{i}$ 10

$\vec{a} = -9.81 \frac{m}{s^2} \hat{j}$

b. Find the height of the cliff.

$0 = H - \frac{1}{2} g (17.5)^2$ 10

$H = 1.50 \times 10^3 \ m$

c. Find the velocity and acceleration of the ball just before it hits the ground in vector notation.

$\vec{a} = -9.81 \frac{m}{s^2} \hat{i}$ 3

$V_y = 0 - 9.81(17.5) = -172 \ m/s$

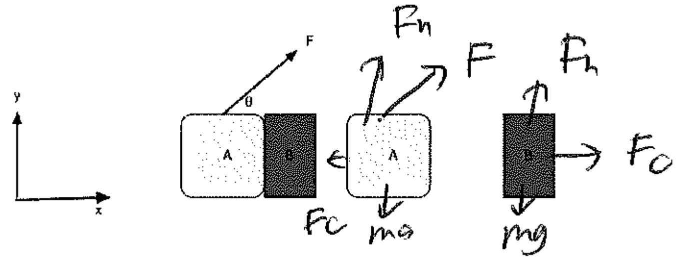
$\vec{v} = (25.5 \hat{i} - 172 \hat{j}) \ m/s$ 7

d. Find the position of the ball at a time of 11.5 s in vector notation.

$x = 25.5(11.5) = 293 \ m$

$y = 1.50 \times 10^3 - \frac{1}{2} g (11.5)^2 = 851 \ m$ 15

$\vec{r} = (293 \hat{i} + 851 \hat{j}) \ m$



4. A block A with mass m_A has a rope attached which is pulled with a force, F, at an angle θ, as shown above. Block A pushes against block B (m_B), both on a frictionless surface.

a. On the figures above and to the right, draw free body diagrams for both blocks. /10

b. Write out Newton's 2nd Law for both blocks in both directions. (Clearly label blocks "A" and "B").

A: x: $F\cos\theta - F_c = m_A a$ ①

y: $F\sin\theta + F_{n_A} - m_A g = 0$ 20

B: x: $F_c = m_B a$ ②

y: $F_{nB} - m_B g = 0$

Assume m_A = 6.50 kg, m_B = 3.00 kg, F =95.0 N, and θ = 35°0

b. Find the magnitude of the acceleration of the blocks and the magnitude of the contact force between the blocks.

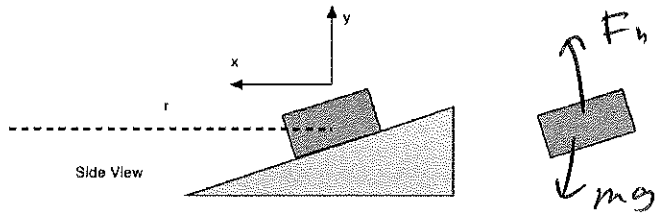
Combine ① and ② !

$$F\cos\theta = (m_A + m_B)a$$ /10

$$a = \frac{(95.0)\cos(35.0^\circ)}{(3.00 + 6.50)} = 8.19\ \frac{m}{s^2}$$

From ②:

$$F_c = (3.00)(8.19) = 24.6\ N$$



5. A 0.750 kg toy car with mass m is traveling at constant speed in a circle of radius r on a frictionless banked track which makes an angle θ with respect to the horizontal direction. The speed of the car is v.

a. On the figure to the right above (side view), draw a free-body diagram of the car. 5

b. Write out Newton's 2nd Law in both directions using the coordinate system in the sketch. Hint: The normal force makes an angle of θ with respect to the y-axis.

x: $F_n \sin\theta = ma = \frac{mv^2}{r}$ 10

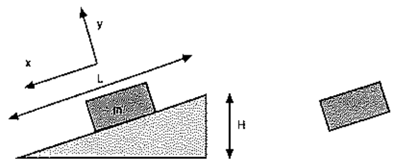
y: $F_n \cos\theta - mg = 0$ 10

Given that m = 0.750 kg, r = 0.850 m, and v = 6.75 m/s.

c. Find the magnitude of the force the keeps the car going in a circle (the "centripetal force").

$$\frac{mv^2}{r} = \frac{(.750)(6.75)^2}{0.850}$$ 10

$$= 40.2\ N$$



6. A block of mass m is at rest at a height H at the top of a frictionless ramp with a length L. The block is then released and slides down the ramp leaving the ramp at the bottom with a speed v. Assume m=4.00 kg, H=15.5m, and L=27.5 m.

a. Using forces and kinematics, find the speed of the block at the bottom of the ramp.

$$\theta = \sin^{-1}\left(\frac{15.5}{27.5}\right) = 34.3^\circ$$

F_x: $\quad mg\sin\theta = ma, \quad a = g\sin\theta$

$$= g\sin 34.3 = 5.53\, m/s^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$v^2 = 0 + 2(5.53)(27.5)$$

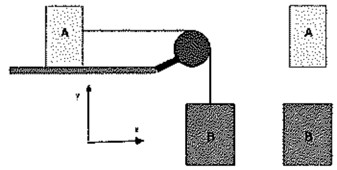
$$v = 17.4\, m/s$$

b. The block then slides along a horizontal, rough floor for a distance of 35.5 m before coming to rest. Use Conservation of Energy to find the coefficient of friction between n block and floor.

$$\frac{1}{2}\cancel{m}v^2 = f_k\Delta s = \mu_k F_n\Delta s = \mu_k \cancel{m}g\,\Delta s$$

$$\mu_k = \frac{v^2}{2g\Delta s}$$

$$= \frac{17.4^2}{2g(35.5)} = 0.437$$



7. Blocks A and B are connected by a light string and attached over a pulley and are initially at rest. Assume a frictionless surface under block A. The blocks have masses m_A and m_B. The pulley is a solid disk with radius R and mass M.The blocks are released.

a. Write out the torque equation for the pulley.

$$(T_B - T_A) R = I\alpha = I\frac{a}{R}$$ 10

Take $m_A = 5.00$ kg, $m_B = 7.00$ kg, $M = 4.00$ kg, and $R = 1.25$ m.

b. Find the moment of inertia of the pulley about its axis.

$$\frac{1}{2}MR^2 = \frac{1}{2}(4.00)(1.25)^2 = 3.12 \text{ kg m}^2$$ 5

c. Using Conservation of Energy, find the speed of the blocks after they have moved 3.00 m.

$$E_i = E_f$$

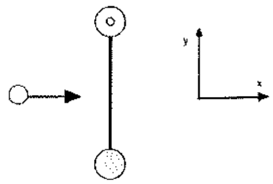
$$m_B g (3.00) = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}I\omega^2$$

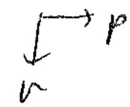
$$= \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}\frac{1}{2}\cancel{MR^2}\frac{v^2}{\cancel{R^2}}$$ 20

$$= \frac{1}{2}v^2\left[m_A + m_B + \frac{M}{2}\right]$$

$$v^2 = \frac{2m_B g(3.00)}{m_A + m_B + \frac{M}{2}} = \frac{6 \cdot 7 \cdot g}{5+7+2}$$

$$v = 5.42 \text{ m/s}$$





8. A barbell consists of a rod of length 1.50 m and mass 0.500 kg and two balls of mass 0.750 kg at the ends of the rod (treat the balls as point masses). The barbell is pinned through one ball and is hanging as shown above. A ball of mass 1.25 kg with a speed of 12.5 m/s traveling in the +x-direction hits the barbell in the middle and sticks to it.

a. Find the linear momentum of the ball in vector notation before the collision.

$$\vec{p} = (1.25)(12.5)\hat{i} = 15.6 \text{ kg}\frac{m}{s}\hat{i}$$ 5

b. Find the angular momentum about the barbell pivot point of the ball in vector notation before the collision. Take "out of the page" as the +z-direction.

$$\vec{L} = \vec{r}\times\vec{p} = (0.750)(15.6)\sin 90^\circ\,\hat{k}$$ 10

$$= 11.7 \text{ kg}\frac{m^2}{s}\hat{k}$$

c. Find the angular momentum about the barbell pivot point of the ball plus barbell in vector notation just after the collision.

Same 5

d. Find the angular speed about the barbell pivot point of the ball plus barbell just after the collision before it swings up.

$$11.7 = I_f\,\omega_f$$ 20

$$I_f = \frac{1}{3}(.500)(1.50)^2 + (.750)(1.50)^2 + 1.25(.750)^2 = 2.77 \text{ kg m}^2$$

$$\omega_f = \frac{11.7}{2.77} = 4.23\ \frac{rad}{s}$$

9. A block of mass 1.75 kg is attached to a spring. The block is stretched and released on a frictionless surface. The equation of motion of the block is given by:

$$x = (1.25m)\cos(12.0t)$$

a. Find the spring constant of the spring.

$\omega = \sqrt{\frac{k}{m}}$, $k = m\omega^2$ 5

$k = (1.75)(12.0)^2 = 252 \frac{N}{m}$

b. Find the total energy of the system

$E = \frac{1}{2} kA^2$

$= \frac{1}{2}(252)(1.25)^2 = 197 J$ 5

c. Find an expression for the velocity of the block as a function of time.

$v = \frac{dx}{dt} = -(12.0)(1.25)\sin(12.0t)$ 10

$= -15.0 \sin(12.0t)$

d. Find the maximum kinetic energy of the block.

$K_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2}(1.75)(15.0)^2$ 5

$= 197 J = E$