Your Name: ____________________

PHY203
Final Exam
5/7/18

- Show work
- Use correct SI units
- Use scientific notation
- All answers with 3 significant figures
- use $g = 9.81\ m/s^2$

Solutions

FinalS18Part1

1. A rocket is launched straight up from rest at ground level with an initial acceleration of 35.0 m/s^2. Take "up" as the positive x-direction and x=0 at ground level. The rocket accelerates for 15.0 s, then the motor cuts off

a. Find the velocity and acceleration of the rocket at its highest point (magnitudes and signs).

$V = 0$ 2

$a = -9.81\ m/s^2$ 2

b. Find the position of the rocket at its highest point.

$x_1 = \frac{1}{2}(35.0)(15.0)^2 = 3938\ m$

$V_1 = (35.0)(15.0) = 525\ m/s$

x_2: $0 = (525)^2 - 2g\Delta x$ 15

$\Delta x = 14{,}048\ m$

$x_2 = 14{,}048 + 3938$

$= 1.80 \times 10^4\ m$

c. Find the total time the rocket is in the air.

t_2: $0 = 525 - g\,t_2$

$t_2 = 53.5\ s$ 15

From top to ground!

$0 = 1.80\times10^4 + 0 - \frac{1}{2} g\, t_3^2$

$t_3 = 60.5\ s$

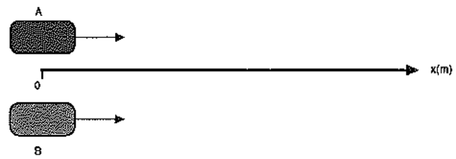
$t = 15.0 + 53.5 + 60.5 = 129\ s$

d. Find the velocity and acceleration (magnitudes and signs) of the rocket just before it hits the ground.

$a = -9.81\ m/s^2$ 2

V: $V = 0 - g\,(60.5)$ from top 4

$V = -594\ m/s$



2. Two trains are traveling to the right on parallel tracks. At a time of t=0 train A passes the x=0 point with a constant speed of 15.0 m/s. At t=2.50 s train B which was at rest at x=0 starts traveling with a constant acceleration of 3.50 m/s^2.

a. Using the coordinate system depicted above, write an equation of motion (x vs. t) for train A:

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$x_A = 15.0\,t$

b. Using the coordinate system depicted above, write an equation of motion (x vs. t) for train B:

$x_B = \frac{1}{2}(3.50)(t-2.50)^2 = 1.75(t-2.50)^2$ 10

c. Find the time at which the centers of the trains are side-by-side.

$$15t = 1.75(t-2.50)^2$$
$$= 1.75(t^2 - 5t + 6.25)$$
$$= 1.75t^2 - 8.75t + 10.9$$
$$0 = 1.75t^2 - 23.8t + 10.9$$
$$t = \frac{23.8 \pm \sqrt{23.8^2 - 4 \cdot 1.75 \cdot 10.9}}{3.50}$$
$$= 13.1\,s,\ \cancel{0.486}\,s$$

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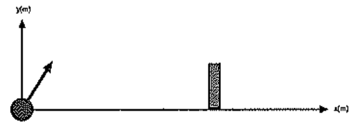

3. A cannonball is shot from ground level at a castle wall. The castle wall is 65.0 m high and a horizontal distance of D = 175 m from the cannon. Assume the cannonball just misses the top of the wall (not at its highest point). The initial horizontal speed of the cannonball is 40.0 m/s

a. Find the time it takes the ball to reach the wall.

$175 = (40.0)t$

$t = 4.38\,s$

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b. Find the initial velocity of the cannonball and write it in vector notation using the coordinate system above.

$v_x = 40.0\ m/s$

v_y: $65.0 = 0 + v_{y0}(4.38) - \frac{1}{2}g(4.38)^2$

$v_{y0} = 36.3\ m/s$

$\vec{v}_0 = (40.0\hat{i} + 36.3\hat{j})\ m/s$

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c. Find the velocity of the cannonball just as it passes over the wall and write it in vector notation.

$v_x = 40.0\ m/s$

v_y: $v_y = 36.3 - g(4.38)$

$= -6.64\ m/s$

$\vec{v} = (40.0\hat{i} - 6.64\hat{j})\ m/s$

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d. Find the velocity and acceleration of the cannonball at its highest point in vector notation.

$\vec{v} = v_{0x}\hat{i} = 40.0\hat{i}\ m/s$

$\vec{a} = -9.81\hat{j}\ m/s^2$

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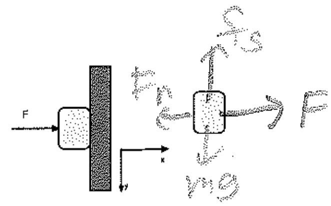
Final2S18



4. A force F is pushing a block of mass m against a <u>rough</u> wall. The block is not moving.

a. Draw a free body diagram of the block on the figure up and to the right. 5

b. Write out Newton's 2nd Law for the block in both directions.

$$x: \quad F - F_n = ma = 0$$ 10

$$y: \quad mg - f_s = ma = 0$$

Take m=7.50 kg and the coefficients of friction between wall and block to be and μ_k =0.350 and μ_s =0.550.

b. Find the minimum force F that will keep the block from falling.

$$f_{s\,max} = \mu_s F_n = \mu_s F$$ 10

$$F = \frac{mg}{\mu_s} = \frac{(7.50)g}{0.550}$$

$$= 134\ N$$

c. For F <u>twice</u> what you found in part b., find the magnitudes of the normal and frictional forces.

$$F_n = F = 2 \cdot 134 = 268\ N$$

$$f_s = mg = (7.50)g = 73.6\ N$$ 5

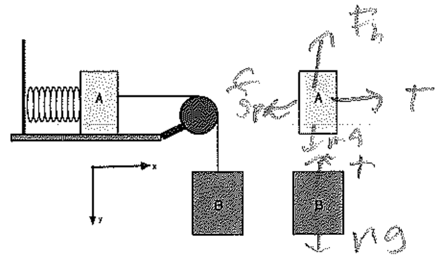
d. For F <u>half</u> what you found in part b., find the magnitude of the acceleration of the block.

$$F_n = F = \frac{134}{2} = 67.0\ N$$

$$mg - f_k = ma$$ 10

$$a = g - \frac{\mu_k F_n}{m}$$

$$= g - \frac{(0.350)(67.0)}{7.50} = 6.68\ m/s^2$$



5. A massless spring is attached to block A which is at rest on a frictionless surface. Block A is attached to a massless string which is passed over a massless and frictionless pulley and attached to block B. The spring with spring constant k is initially uncompressed. The blocks are released and start to move.

a. Draw free body diagrams on the blocks (while they are moving) above and to the right.

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b. Write out Newton's 2nd Law for both blocks in both directions.

A: x: $T - kx = ma$

y: $m_A g - F_n = 0$ 9

B: y: $m_B g - T = m_B a$

Given M_A=5.50 kg, M_B=17.5 kg, and k=85.0 N/m,

c. Find the speed of the blocks after they have traveled 2.50 m. Take y=2.50 m as the initial height of block B.

Use Conservation of Energy:

$$(17.5)g(2.50) = \frac{1}{2}(5.50+17.5)v^2 + \frac{1}{2}(85.0)(2.50)^2$$

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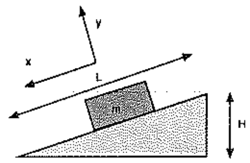
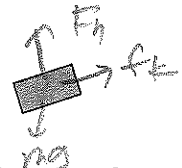
$$429 = \frac{1}{2}(23)v^2 + 266$$

$$v = 3.77 \text{ m/s}$$





6. A block of mass m is at rest at a height H at the top of a rough ramp with a length L. The block is then released and slides down the ramp leaving the ramp at the bottom with a speed v. The coefficient of kinetic friction between block and ramp is μ_k.

a. Above and to the right draw a free body diagram of the block while it is sliding down the ramp. 4

b. Write out Newton's 2nd Law for the block in both directions while it is sliding on the ramp.

x: $mg\sin\theta - f_k = ma$ 3

y: $F_n - mg\cos\theta = 0$ 3

Assume m=4.00 kg, H=15.5m, L=27.5 m, and v=12.5 m/s.

c. Find the magnitude of the normal force on the block.

$F_n = (4.00)g\cos\theta$ $\theta = \sin^{-1}\left(\frac{15.5}{27.5}\right)$ 5

$= 32.4$ N $= 34.3°$

d. Using energy conservation, find the coefficient of friction, μ_k.

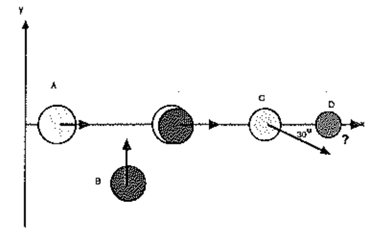
$\cancel{m}gH = \frac{1}{2}mv^2 + \mu_k F_n L$

$= \frac{1}{2}\cancel{m}v^2 + \mu_k \cancel{m}g\cos(34.3°)L$ 15

$g(15.5) - \frac{1}{2}(12.5)^2 = \mu_k g\cos(34.3°)(27.5)$

$\mu_k = 0.332$

Final3S18



7. Two pucks traveling on a frictionless surface collide inelastically but do not stick together. Before the collision puck A (mass of 4.50 kg) is traveling in the positive x-direction with a speed of 6.50 m/s, and puck B (mass of 3.50 kg) is traveling in the positive y-direction with a speed of 5.50 m/s. After the collision, two new pucks are formed, Puck C has a mass of 5.00 kg and is traveling with a speed of 6.00 m/s in a direction that is -30° with respect to the x-axis, as shown above. Puck D is the other puck after the collision.

a. Find the initial momenta of pucks A and B and write them in vector notation.

$\vec{p_A} = (4.50)(6.50)\hat{i} = 29.2\,\hat{i}\ kg\frac{m}{s}$ 5

$\vec{p_B} = (3.50)(5.50)\hat{j} = 19.2\,\hat{j}\ kg\frac{m}{s}$ 5

b. Find the final momentum of puck C and write it in vector notation.

$\vec{p_C} = (5.00)(6.00)(\cos 30^\circ\,\hat{i} - \sin 30^\circ\,\hat{j})$ 10

$= (26.0\,\hat{i} - 15\hat{j})\ kg\frac{m}{s}$

c. Find the final velocity of puck D and write it in vector notation.

x: $29.2 = 26.0 + m_D v_{xD}$

$= 26.0 + (3.00)v_{xD}$ 15

$v_x = 1.07\ m/s$

y: $19.2 = -15.0 + (3.00)v_{yD}$

$v_y = 11.4\ m/s$

$\vec{v_D} = (1.07\,\hat{i} + 11.4\,\hat{j})\ m/s$

8. A wheel is at rest at the top of a rough ramp which makes an angle of 35.0° with respect to the horizontal. The height of the ramp is 3.50 m. The wheel consists of a thin rim with mass 2.50 kg at a radius of 1.50 m and 4 spokes of length 1.50 m and mass 0.350 kg apiece. Assume the wheel rolls without slipping down the ramp.

a. Find the moment of inertia of the wheel about its axis.

$$I = (2.50)(1.50)^2 + 4\left(\frac{1}{3}\right)(.350)(1.50)^2$$ 5

$$= 6.47 \text{ kg m}^2$$

b. Sketch a free body diagram of the wheel on the ramp.

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c. Use Conservation of Energy to find the linear speed of the wheel at the bottom of the ramp.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}v^2\left[m + \frac{I}{r^2}\right]$$ 20

$$m = 2.50 + 4(.350) = 3.90 \text{ kg}$$

$$(3.90)g(3.50) = \frac{1}{2}v^2\left[3.90 + \frac{6.47}{1.50^2}\right]$$

$$v = 6.25 \text{ m/s}$$

9. A block of mass 1.75 kg is attached to a spring. The block is stretched and released on a frictionless, horizontal surface. The equation of motion of the block is given by:

$$x = (0.750\,m)\cos(3.50t)$$

a. Find the spring constant of the spring.

$\omega = \sqrt{\frac{k}{m}}$, $k = m\omega^2 = (1,75)(3,50)^2$

$= 21,4$ N/m

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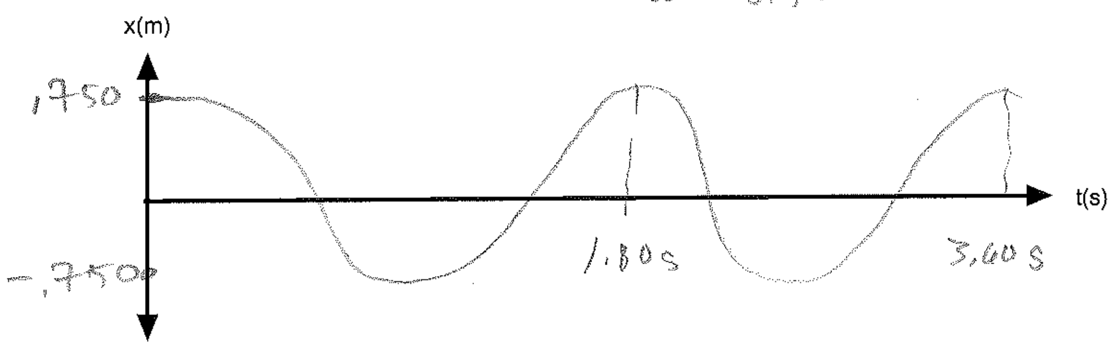
b. Make a plot of position vs. time for the first 2 periods. Add values to the plot below (on both the x and t axes). $T = \frac{2\pi}{\omega} = \frac{2\pi}{3,50} = 1,80$ s



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c. Find an expression for the velocity of the block as a function of time.

$v = \frac{dx}{dt} = -(3,50)(,750)\sin(3,50)t$

$= -2,62 \sin(3,50)t$

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d. Find the maximum speed of the block.

2,62 m/s

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e. Find the velocity of the block at a time of 1.75 s.

$v = -2,62 \sin(3,50)(1,75)$

$= 0,413 \frac{m}{s}$

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