Your Name: ______________________

PHY203
Final Exam
12/14/18

- Show work
- Use correct SI units
- Use scientific notation
- All answers with 3 significant figures
- use $g = 9.81\ m/s^2$

Solutions

1. A motorcycle accelerates from rest in the +x direction at 5.00 m/s^2 for a time of 22.5 s. It then travels at constant speed for 55.0 s. Finally it slows down until coming to rest with an acceleration of magnitude 3.50 m/s^2.

a. Find the speed of the motorcycle after 22.5 s.

$V = 0 + (5.00)(22.5) = 112 \text{ m/s}$ 5

b. Find the distance the motorcycle has traveled after 22.5 s.

$x = 0 + 0 + \frac{1}{2}(5.00)(22.5)^2$

$= 1.27 \times 10^3 \text{ m}$ 5

c. Find the total distance the motorcycle has traveled after 77.5 s.

$x = 1.27 \times 10^3 + (112)(55.0)$

$= 7.43 \times 10^3 \text{ m}$ 5

d. Find the total time the motorcycle has been traveling before it comes to rest.

t_3: $0 = 112 - 3.50t$

$t = 32.0$ 10

$t = 77.5 + 32.0 = 110 \text{ s}$

e. Find the total distance the motorcycle has traveled before it comes to rest.

$x = 7.43 \times 10^3 + 112(32.0) - \frac{1}{2}(3.50)(32.0)^2$

$= 9.22 \times 10^3 \text{ m}$ 10

2. Given an initial position, initial velocity, and (constant) acceleration of a ball:

$x_o = (6.50\hat{\imath} - 8.50\hat{\jmath})m$
$v_o = (-5.00\hat{\imath} + 7.00\hat{\jmath})m/s$
$a_0 = (4.50\hat{\imath} - 3.50)m/s^2$

a. Find the magnitude of the initial velocity and the angle the velocity vector makes with the +x-axis.

$V_o = \sqrt{(-5.00)^2 + (7.00)^2} = 8.60 \frac{m}{s}$ 5

$\tan^{-1}\left(\frac{7}{+5}\right) = 54.5^\circ$

$\theta = 180 - 54.5 = 126^\circ$ 5

b. Find the acceleration, velocity, and position of the ball in vector notation at a time of 9.50 s.

$\vec{a} = \vec{a}_o = (4.50\hat{i} - 3.50\hat{j})\ m/s$ 5

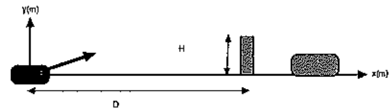
$\vec{V} = \vec{V}_o + \vec{a}t = (-5.00\hat{i} + 7.00\hat{j})$
$+ (4.50\hat{i} - 3.50\hat{j})(9.50)$ 5

$= (37.8\hat{i} - 26.2\hat{j})\ m/s$

$\vec{X} = \vec{X}_o + \vec{V}_o t + \frac{1}{2}\vec{a}t^2$

$= (6.50\hat{i} - 8.50\hat{j}) + (-5.00\hat{i} + 7.00\hat{j})(9.50)$
$+ \frac{1}{2}(4.50\hat{i} - 3.50\hat{j})(9.50)^2$ 5

$= (162\hat{i} - 99.9\hat{j})\ m$



3. A cannonball is shot from ground level at a castle wall. The initial horizontal speed of the ball is 45.0 m/s ~~at~~ at angle of 35.0° with respect to the horizontal direction. The castle wall has height H and horizontal distance D = 250 m from the cannon.

a. Write the initial velocity of the cannonball in vector notation using the coordinate system above.

$$\tan\theta = \frac{v_{y0}}{v_{x0}}, \quad \tan 35^\circ = \frac{v_{y0}}{45.0}$$ 10

$$v_{y0} = 31.5 \text{ m/s}$$

$$\vec{v_0} = (45.0\hat{i} + 31.5\hat{j}) \text{ m/s}$$

b. Write the velocity and acceleration of the cannonball at its highest point in vector notation.

$$\vec{a} = -9.81\hat{j} \text{ m/s}^2$$ 10

$$\vec{v} = 45.0\hat{i} \text{ m/s}$$

c. Find the height of the cannonball at its highest point.

$$0 = (31.5)^2 - 2g\Delta y$$ 5

$$\Delta y = H = 50.6 \text{ m}$$

Assume the cannonball just grazes the top of the wall (but not at its highest point).

d. Find the time it takes the ball to reach the wall.

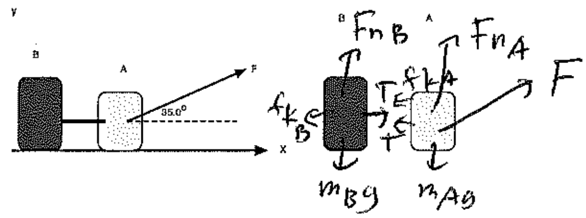
$$250 = 0 + 45.0t$$ 5

$$t = 5.56 \text{ s}$$

e. Find the height of the castle wall.

$$y = 0 + 31.5(5.56) - \tfrac{1}{2}g(5.56)^2$$ 10

$$= 23.8 \text{ m}$$



4. A block A with mass 7.50 kg is pulled with a force of F = 55.0 N at an angle of θ = 35.0°, as shown. Block A is attached by a horizontal massless string to block B (mass of 9.00 kg). Take the same coefficient of kinetic friction for both blocks to be μ_k =0.200.

a. Draw free-body diagrams for block A and block B on the diagrams to the right above. 5

b. Write out Newton's 2nd Law for both blocks in the x- and y-directions.

A: x: $55.0\cos 35.0° - T - f_{kA} = m_A a$

y: $55.0\sin 35.0° + F_{NA} - m_A g = 0$ 20

B: x: $T - f_{kB} = m_B a$

y: $F_{nB} - m_B g = 0$

c. Find the magnitude of the normal forces on the blocks. 42.0 N

A: $F_{NA} = (7.50)g - 55.0\sin 35.0 =$ 4

B: $F_{NB} = (9.00)g = 88.3$ N

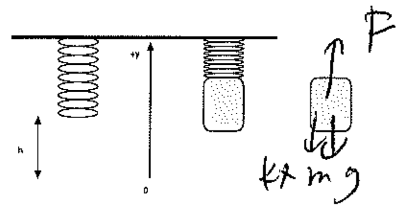
d. Find the magnitude of the acceleration of the blocks.

$55.0\cos 35.0 - T - (.200)(42.0) = m_A a$

$T - (.200)(88.3) = m_B a$ 6

$45.1 - 8.5 - 17.7 = (7.50 + 9.00)a$

$a = 1.15\ m/s^2$



5. A massless spring is hanging from the ceiling. The bottom of the spring is a distance h from ground level (y=0). A block of mass M is <u>attached</u> to the spring and the block is pushed by a force F until the spring is compressed by a distance d. The spring constant is k.

a. Draw a free body diagram on the block above and to the right. 5

b. Write out Newton's 2nd Law for the block in the y direction.

$$F - mg - kx = 0$$ 5

Given M=15.5 kg, h = 3.50 m, d=2.35 m, and k = 200 N/m.

c. Find the magnitude of F. 5

$$F = (15.5)g + (2.35)(200) = 622\,N$$

d. The block is released. Find the total energy of the block just after it is released. (Take y=0 at ground level.)

$$E = mgH + \frac{1}{2}kx^2$$

$$= (15.5)g(3.50+2.35) + \frac{1}{2}(200)(2.35)^2$$ 10

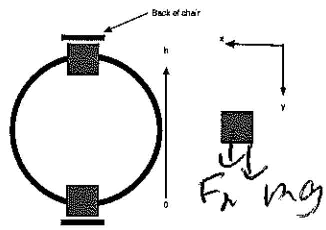
$$= 1.44 \times 10^3\,J$$

e. Find the speed of the block just before it hits the ground. (Note that the spring remains attached to the block as it falls.)

$$1.44 \times 10^3 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}(15.5)v^2 + \frac{1}{2}(200)(3.5)^2$$ 10

$$v = 5.27\,m/s$$



6. A ferris wheel has a radius of 13.5 m. At the bottom of the trajectory the speed of a person on the ferris wheel is 28.0 m/s. Take h=0 at the bottom of the wheel as shown above. Assume when the ferris wheel takes the person to the top that the energy of the passenger (m=75.0 kg) is conserved. Assume the chair rotates so the back of the chair is always away from the center.

a. Find the energy of the passenger at the bottom of the trajectory.

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(75.0)(28.0)^2 = 2.94\times10^4\ \text{J}$$ 5

b. Find the speed of the passenger at the top of the trajectory.

$$2.94\times10^4 = \frac{1}{2}mv^2 + mgH$$

$$= \frac{1}{2}(75.0)v^2 + (75.0)g(27.0)$$ 10

$$v = 15.9\ \text{m/s}$$

c. Draw a free body diagram of the passenger at the top on the figure above and to the right. (Call the force of back of chair on passenger to be the normal force.) 5

d. Find the magnitude of the normal force of the back of the chair pushing on the passenger at the top.

$$F_n + mg = ma = \frac{mv^2}{r}$$

$$F_n = (75.0)\left[\frac{15.9^2}{13.5} - g\right]$$ 10

$$= 669\ \text{N}$$

7. A block (A) of mass 5.50 kg is traveling on a frictionless surface in the +x-direction with a speed of 12.5 m/s. it catches up to and collides with a block (B) of mass 8.50 kg that was traveling in the +x-direction with a speed of 9.50 m/s before the collision. After the collision, a block (C) of mass 6.00 kg flies off at an angle of 55.0° with respect to the x-axis with a speed of 13.5 m/s. Block D (mass of 8.00 kg) flies off as well.

a. Write the momenta of blocks A and B before the collision and block C after the collision in vector notation.

$\vec{p}_A = (5.50)(12.5)\hat{i} = 68.8\ kg\frac{m}{s}\ \hat{i}$ 5

$\vec{p}_B = (8.50)(9.50)\hat{i} = 80.8\ kg\frac{m}{s}\ \hat{i}$ 5

$\vec{p}_C = (6.00)(13.5)(\cos 55.0\,\hat{i} + \sin 55.0\,\hat{j})$ 10

$= (46.5\hat{i} + 66.4\hat{j})\ kg\frac{m}{s}$

b. Find the total momentum of the system in vector notation after the collision.

$(68.8 + 80.8)\hat{i} = 150\hat{i}\ kg\frac{m}{s}$ 6

c. Find the velocity in vector notation of block D after the collision.

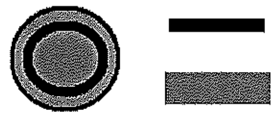
$x:\quad 150 = 46.5 + (8.00)v_x$

$v_x = 12.9\ m/s$ 14

$y:\quad 0 = 66.4 + (8.00)v_y$

$v_y = -8.30\ m/s$

$\vec{v}_D = (12.9\hat{i} - 8.30\hat{j})\ m/s$



Top View Side View

8. A solid disk of mass 7.50 kg and radius 2.50 m is rotating at a constant angular speed of 25.0 rad/s in a clockwise direction as viewed from above.

a. Find the moment of inertia and the angular momentum of the disk in vector notation taking the +z-direction as "up" and perpendicular to the disk.

$I = \frac{1}{2}(7.50)(2.50)^2 = 23.4 \text{ kg m}^2$ 5

$\vec{L} = -(23.4)(25)\hat{k}$

$= -586\hat{k} \text{ kg} \frac{m^2}{s}$ 5

b. Find the kinetic energy of the rotating disk.

$K = \frac{1}{2}(23.4)(25.0)^2 = 7.31 \times 10^3 \text{ J}$ 5

A thin ring of mass 5.50 kg and radius 2.10 m is dropped from a height of 1.50 m onto the disk such that the ring and disk are centered on each other. The ring and disk stick together.

c. Find the angular momentum of the ring/disk system in vector notation after the collision.

$\vec{L}_i = \vec{L}_f = -586\hat{k} \text{ kg} \frac{m^2}{s}$ 5

d. Find the angular speed of the ring/disk system after the collision.

$586 = (23.4 + mr^2)\omega_f$ 10

$= (23.4 + 24.3)\omega_f$

$\omega_f = 12.3 \frac{rad}{s}$

$(5.50)(2.1)^2 = 24.3$

9. A block of mass 1.75 kg is attached to a spring and resting on a horizontal, frictionless table. The block is pulled horizontally such that the spring is stretched by 35.0 cm and released at t=0. The period of the block's motion is 0.450 s.

a. Find the angular frequency of the block's motion.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.450} = 14.0 \ \frac{rad}{s}$$

5

b. Write an equation of motion of the block.

$$k = m\omega^2 = (1.75)(14.0)^2 = 341$$

$$x = (0.350)\cos(14.0t) \ m$$

10

c. Find the energy of the block.

$$E = \frac{1}{2} k A^2$$

$$= \frac{1}{2}(341)(.350)^2 = 20.9 \ J$$

5

d. Find the velocity the first time the block is at a position of -0.200 m.

$$v = -(14.0)(0.350)\sin(14.0t)$$

$$= -4.90 \sin(14.0t)$$

10

$$-0.200 = (0.350)\cos(14.0t)$$

$$t = 0.156 \ s$$

$$v = -4.90\sin(14.0 \cdot 0.156)$$

$$= -4.02 \ m/s$$